

Supplementary Material for “Self-adaptive Bat ptive Bat Algorithm With Genetic Operations”

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TABLE I
MULTIMODAL BENCHMARK FUNCTIONS (F_8 – F_{13})

Function	D	Range	$\check{f}$
$F_8(x) = \sum_{i=1}^n [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]$	0
$F_9(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$	30	$[-32, 32]$	0
$F_{10}(x) = \frac{1}{4000}\sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	$[-600, 600]$	0
$F_{11}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\} + \sum_{i=1}^n u(x_i, 5, 100, 4), y_i = 1 + \frac{x_i + 1}{4}$	30	$[-10, 10]$	0
$F_{12}(x) = 0.1\left\{\sin^2(3\pi x_1) + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_{i+1})] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)]\right\} + \sum_{i=1}^n u(x_i, 5, 100, 4)$	30	$[-5, 5]$	0
$F_{13}(z) = \sum_{i=1}^n [z_i^2 - 10\cos(2\pi z_i) + 10], z = \Lambda^{10} T_{asy}^{0.2}(T_{osc}(x - x^{opt}))$	30	$[-5, 5]$	0

TABLE II
FIXED-DIMENSION MULTIMODAL BENCHMARK FUNCTIONS (F_{14} – F_{23})

Function	D	Range	$\check{f}$
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6}\right)^{-1}$	2	$[-65, 65]$	1
$F_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_i(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	$[-5, 5]$	0.0003
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^4 + 4x_2^4$	2	$[-1, 1]$	-1.0316
$F_{17}(x) = \left(x_2 - \frac{5}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	2	$[-5, 5]$	0.398
$F_{18}(x) = [1 + (x_1 + x_2 + 1)^2 [19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2]] \times [30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-2, 2]$	3
$F_{19}(x) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^3 a_{ij}(x_j - p_{ij})^2\right)$	3	$[1, 3]$	-3.86
$F_{20}(x) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^6 a_{ij}(x_j - p_{ij})^2\right)$	6	$[0, 1]$	-3.32
$F_{21}(x) = -\sum_{i=1}^5 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.1532
$F_{22}(x) = -\sum_{i=1}^7 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.4028
$F_{23}(x) = -\sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.5363

TABLE III
COMPOSITE BENCHMARK FUNCTIONS (F_{24} – F_{29})

Function	D	Range	$\tilde{f}$
$F_{24}(x)$ (composition function 1): $f_1, f_2, f_3, \dots, f_{10}$ = Sphere Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (1, 1, 1, \dots, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (\frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \dots, \frac{1}{20})$	10	[-5,5]	0
$F_{25}(x)$ (composition function 2): $f_1, f_2, f_3, \dots, f_{10}$ = Griewank's Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (1, 1, 1, \dots, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (\frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \dots, \frac{1}{20})$	10	[-5,5]	0
$F_{26}(x)$ (composition function 3): $f_1, f_2, f_3, \dots, f_{10}$ = Griewank's Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (1, 1, 1, \dots, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (1, 1, 1, \dots, 1)$	10	[-5,5]	0
$F_{27}(x)$ (composition function 4): f_1, f_2 = Ackley's Function f_3, f_4 = Rastrigin's Function f_5, f_6 = Weierstras's Function f_7, f_8 = Griewank's Function f_9, f_{10} = Sphere Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (1, 1, 1, \dots, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (\frac{5}{32}, \frac{5}{32}, 1, 1, 10, 10, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20})$	10	[-5,5]	0
$F_{28}(x)$ (composition function 5): f_1, f_2 = Rastrigin's Function f_3, f_4 = Weierstras's Function f_5, f_6 = Griewank's Function f_7, f_8 = Ackley's Function f_9, f_{10} = Sphere Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (1, 1, 1, \dots, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (\frac{1}{5}, \frac{1}{5}, 10, 10, \frac{1}{20}, \frac{1}{20}, \frac{5}{32}, \frac{5}{32}, \frac{1}{20}, \frac{1}{20})$	10	[-5,5]	0
$F_{29}(x)$ (composition function 6): f_1, f_2 = Rastrigin's Function f_3, f_4 = Weierstras's Function f_5, f_6 = Griewank's Function f_7, f_8 = Ackley's Function f_9, f_{10} = Sphere Function $(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}) = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)$ $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}) = (\frac{1}{50}, \frac{1}{25}, 3, 4, \frac{1}{40}, \frac{3}{100}, \frac{7}{64}, \frac{1}{8}, \frac{9}{200}, \frac{1}{20})$	10	[-5,5]	0

TABLE IV
RESULTS OF MULTIMODAL BENCHMARK FUNCTIONS (F_8 – F_{13})
(CONTINUED)

Function	Algorithm	Avg.	Std.
F_9	SBAGO	0.2678	0.5099
	GA	0.6253	0.5814
	BA	15.5818	0.9581
	SA	20.3377	0.2407
	PSO	9.7055	1.2284
	SAPSO	11.0694	0.9969
	SGABA	13.9842	3.0248
	GBA	3.1342	2.584
	AGPSO	4.0541	0.9227
F_{10}	PSOGSA	13.6588	5.2386
	SBAGO	0.0228	0.0240
	GA	0.2609	0.3621
	BA	152.891	47.4349
	SA	2.052	0.2045
	PSO	0.9908	0.1057
	SAPSO	11.0694	0.9969
	SGABA	43.6731	13.759
	GBA	0.8473	0.6858
F_{11}	AGPSO	0.1268	0.0912
	PSOGSA	25.4325	46.7533
	SBAGO	1.8601×10⁻⁷	0.0189
	GA	0.6842	0.8414
	BA	8.2548×10 ⁶	1.5789×10 ⁷
	SA	1.7833	0.6837
	PSO	1133.3809	5100.455
	SAPSO	8.3681×10 ⁴	2.2601×10 ⁵
	SGABA	1.5236	0.5143
F_{12}	GBA	0.0025	0.0049
	AGPSO	0.2163	0.3565
	PSOGSA	1.7399	1.2418
	SBAGO	0.0029	0.0049
	GA	1.4835	1.6487
	BA	2.9601×10 ⁷	2.4155×10 ⁷
	SA	0.9425	0.5601
	PSO	2.2135×10 ⁴	4.3390×10 ⁴
	SAPSO	2.3121×10 ⁶	2.5738×10 ⁶
F_{13}	SGABA	5.0456	5.0476
	GBA	0.1331	0.5387
	AGPSO	0.0501	0.0605
	PSOGSA	0.675	1.0892
	SBAGO	9.0258	3.4038
	GA	25.3738	9.179
	BA	212.0514	65.9347
	SA	611.0421	130.4859
	PSO	357.6235	59.2004
F_8	SAPSO	198.7541	35.1731
	SGABA	345.6713	141.3983
	GBA	19.3087	50.9449
	AGPSO	119.0298	30.8216
	PSOGSA	223.8829	56.4964

TABLE IV
RESULTS OF MULTIMODAL BENCHMARK FUNCTIONS (F_8 – F_{13})

Function	Algorithm	Avg.	Std.
F_8	SBAGO	5.8329	2.5907
	GA	8.374	12.6965
	BA	76.32	30.038
	SA	242.109	43.3963
	PSO	208.6025	32.5851
	SAPSO	135.99	26.2069
	SGABA	271.9708	59.3021
	GBA	28.325	34.1519
	AGPSO	69.0238	19.0081
	PSOGSA	151.7044	37.5043

TABLE V

RESULTS OF FIXED-DIMENSION MULTIMODAL FUNCTIONS (F_{14} – F_{18})

Function	Algorithm	Avg.	Std.
F_{14}	SBAGO	0.998	7.8179×10^{-16}
	GA	496.3288	2.8908×10^{-13}
	BA	10.2739	7.3158
	SA	57.2715	148.6451
	PSO	5.0885	5.3977
	SAPSO	3.8456	3.5002
	SGABA	10.1463	6.2947
	GBA	79.2195	151.9221
	AGPSO	0.998	1.6493×10^{-16}
	PSOGSA	5.3631	4.726
F_{15}	SBAGO	0.0003	0.0093
	GA	0.3466	0.5644
	BA	0.0056	0.0082
	SA	0.1433	0.2631
	PSO	0.0062	0.0078
	SAPSO	0.0062	0.0121
	SGABA	0.0223	0.0343
	GBA	0.0025	0.0044
	AGPSO	0.0021	0.005
	PSOGSA	0.0107	0.0128
F_{16}	SBAGO	−1.0316	9.1901×10^{-10}
	GA	24.5828	7.2269×10^{-15}
	BA	−0.8684	0.3321
	SA	−0.5805	0.8844
	PSO	−1.0316	6.5843×10^{-16}
	SAPSO	−1.0316	6.5195×10^{-16}
	SGABA	−1.0316	1.598×10^{-7}
	GBA	−1.0269	0.0196
	AGPSO	−1.0316	5.7578×10^{-16}
	PSOGSA	−1.0316	5.6835×10^{-16}
F_{17}	SBAGO	0.3979	5.3169×10^{-16}
	GA	20.6021	0.0196
	BA	0.3979	1.073×10^{-9}
	SA	41.0809	44.1755
	PSO	0.3979	0
	SAPSO	0.3979	0
	SGABA	0.0196	0.0027
	GBA	3.9997	3.8598
	AGPSO	0.3979	0
	PSOGSA	0.3979	0
F_{18}	SBAGO	3	3.2775×10^{-8}
	GA	195.5885	0
	BA	7.5	15.9865
	SA	132.5185	283.2488
	PSO	5.7	14.7885
	SAPSO	8.4	20.5504
	SGABA	4.8	6.8501
	GBA	29.0078	17.4606
	AGPSO	3	2.7868×10^{-15}
	PSOGSA	5.7	14.7885

TABLE VI

RESULTS OF FIXED-DIMENSION MULTIMODAL FUNCTIONS (F_{19} – F_{23})

Function	Algorithm	Avg.	Std.
F_{19}	SBAGO	−3.8628	5.1682×10^{-8}
	GA	−1.8296	9.0336×10^{-16}
	BA	−3.837	0.1411
	SA	−2.0018	1.1535
	PSO	3.8609	0.0034
	SAPSO	−3.837	0.1411
	SGABA	−3.7178	0.2562
	GBA	−3.7922	0.0503
	AGPSO	−3.8628	2.5684×10^{-15}
	PSOGSA	−3.8625	0.0014
F_{20}	SBAGO	−3.2903	0.0535
	GA	−1.1674	9.0336×10^{-16}
	BA	−3.2744	0.0593
	SA	−0.1636	0.1908
	PSO	−3.0708	0.3623
	SAPSO	−3.2114	0.3073
	SGABA	−3.0198	0.2
	GBA	−3.2459	0.036
	AGPSO	−3.2602	0.064
	PSOGSA	−3.2587	0.0986
F_{21}	SBAGO	−9.9848	0.9224
	GA	−9.9848	0.9224
	BA	−5.4904	3.6354
	SA	−0.1980	0.0604
	PSO	−6.5637	3.5148
	SAPSO	−5.8147	3.4673
	SGABA	−5.3127	3.2242
	GBA	−8.4425	2.7144
	AGPSO	−6.6424	3.4533
	PSOGSA	−5.3102	3.3413
F_{22}	SBAGO	−10.4029	4.9708×10^{-6}
	GA	−9.8754	1.6093
	BA	−6.1955	3.5768
	SA	−0.2857	0.1482
	PSO	−6.3552	3.6893
	SAPSO	−7.1227	3.8191
	SGABA	−6.0416	3.6533
	GBA	−8.0367	3.0224
	AGPSO	−7.3441	3.6038
	PSOGSA	−5.0475	3.1596
F_{23}	SBAGO	−10.3577	0.9787
	GA	−10.1021	1.6799
	BA	−5.0046	3.4737
	SA	−0.3835	0.2891
	PSO	−6.5003	3.8826
	SAPSO	−6.4564	3.6693
	SGABA	−5.3448	3.4275
	GBA	−8.494	2.7605
	AGPSO	−8.078	3.5522
	PSOGSA	−4.9486	3.254

TABLE VII
RESULTS OF COMPOSITE BENCHMARK FUNCTIONS ($F_{24}-F_{29}$)

Function	Algorithm	Avg.	Std.
F_{24}	SBAGO	5.174×10^{-5}	2.1353×10^{-5}
	GA	841.6987	3.8058
	BA	134.3471	122.0234
	SA	183.4015	148.7538
	PSO	227.9845	126.4324
	SAPSO	230.4265	91.0854
	SGABA	405.5984	130.8804
	GBA	649.8908	145.1542
	AGPSO	161.7563	94.3163
	PSOGSA	58.8055	83.7807
F_{25}	SBAGO	31.1713	12.4730
	GA	829.9333	2.6465
	BA	404.9119	172.2077
	SA	628.7704	278.5525
	PSO	225.8441	138.8914
	SAPSO	335.67	133.4766
	SGABA	299.7784	183.2273
	GBA	633.8977	137.0278
	AGPSO	217.23	132.8512
	PSOGSA	206.8121	124.7415
F_{26}	SBAGO	113.5673	21.4738
	GA	901.5394	3.2083
	BA	652.6871	199.4304
	SA	1109.1468	230.8423
	PSO	380.194	116.0284
	SAPSO	539.3548	201.6089
	SGABA	978.3718	218.6981
	GBA	901.8441	23.7013
	AGPSO	257.9615	90.1339
	PSOGSA	467.4577	206.3712

TABLE VII
RESULTS OF COMPOSITE BENCHMARK FUNCTIONS ($F_{24}-F_{29}$)
(CONTINUED)

Function	Algorithm	Avg.	Std.
F_{27}	SBAGO	313.6329	22.099
	GA	904.2227	8.9564
	BA	730.9714	166.5196
	SA	905.6271	169.1093
	PSO	578.8533	197.2932
	SAPSO	629.3885	160.1028
	SGABA	805.1325	169.5468
	GBA	907.7875	36.5546
	AGPSO	449.0567	165.0568
	PSOGSA	582.5263	143.166
F_{28}	SBAGO	25.0829	14.8808
	GA	900.0633	0.0836
	BA	519.7691	219.1269
	SA	334.874	298.8468
	PSO	350.4399	193.0162
	SAPSO	329.7723	160.2844
	SGABA	449.1502	209.668
	GBA	902.5249	10.7271
	AGPSO	185.7063	148.3656
	PSOGSA	154.3424	195.21
F_{29}	SBAGO	500.4639	6.0117
	GA	900.0616	0.1801
	BA	853.5612	125.6071
	SA	862.8846	132.4826
	PSO	842.873	140.3207
	SAPSO	850.9663	127.9743
	SGABA	882.1108	80.2336
	GBA	900.0697	0.1993
	AGPSO	815.3479	185.6837
	PSOGSA	855.0226	120.0762

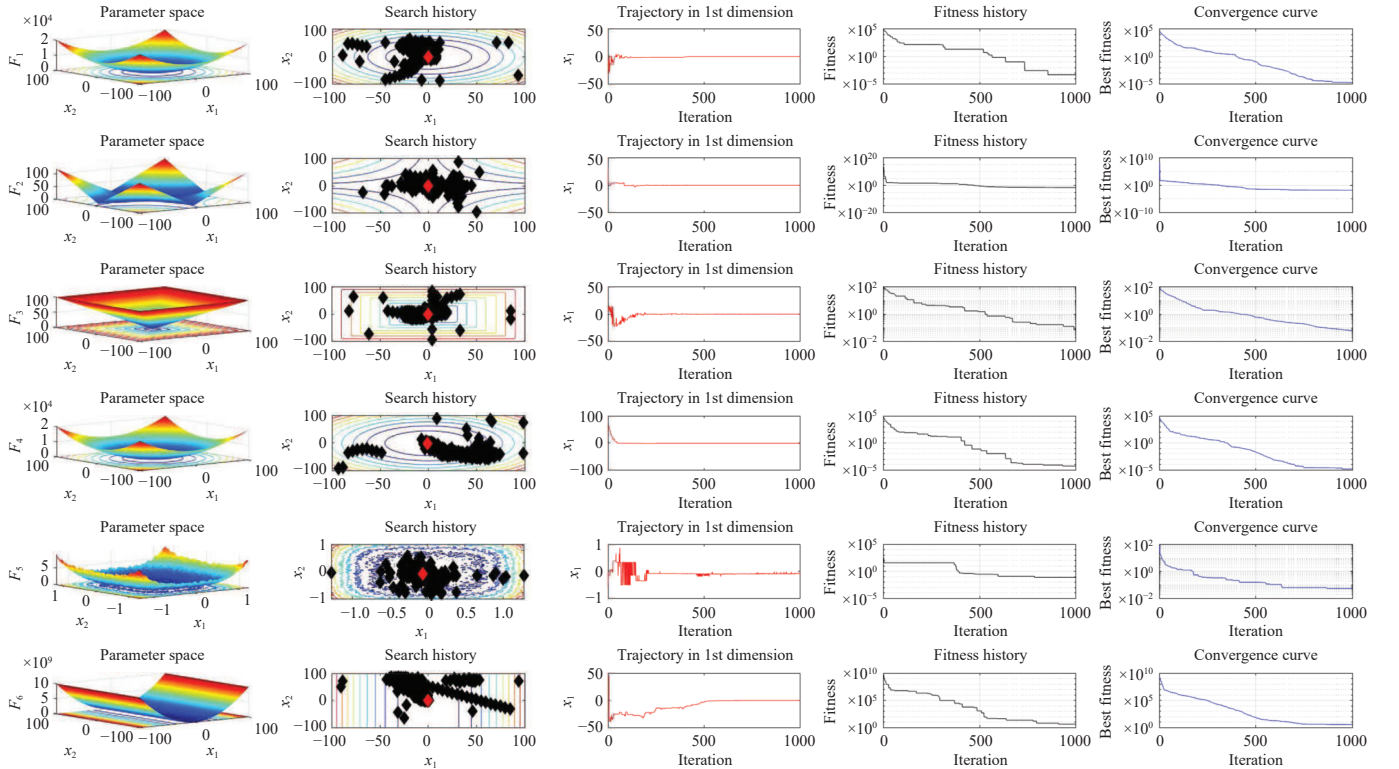


Fig. 1. 2D shapes, search histories, trajectories, fitness histories and convergence curves of F_1-F_6 with SBAGO.

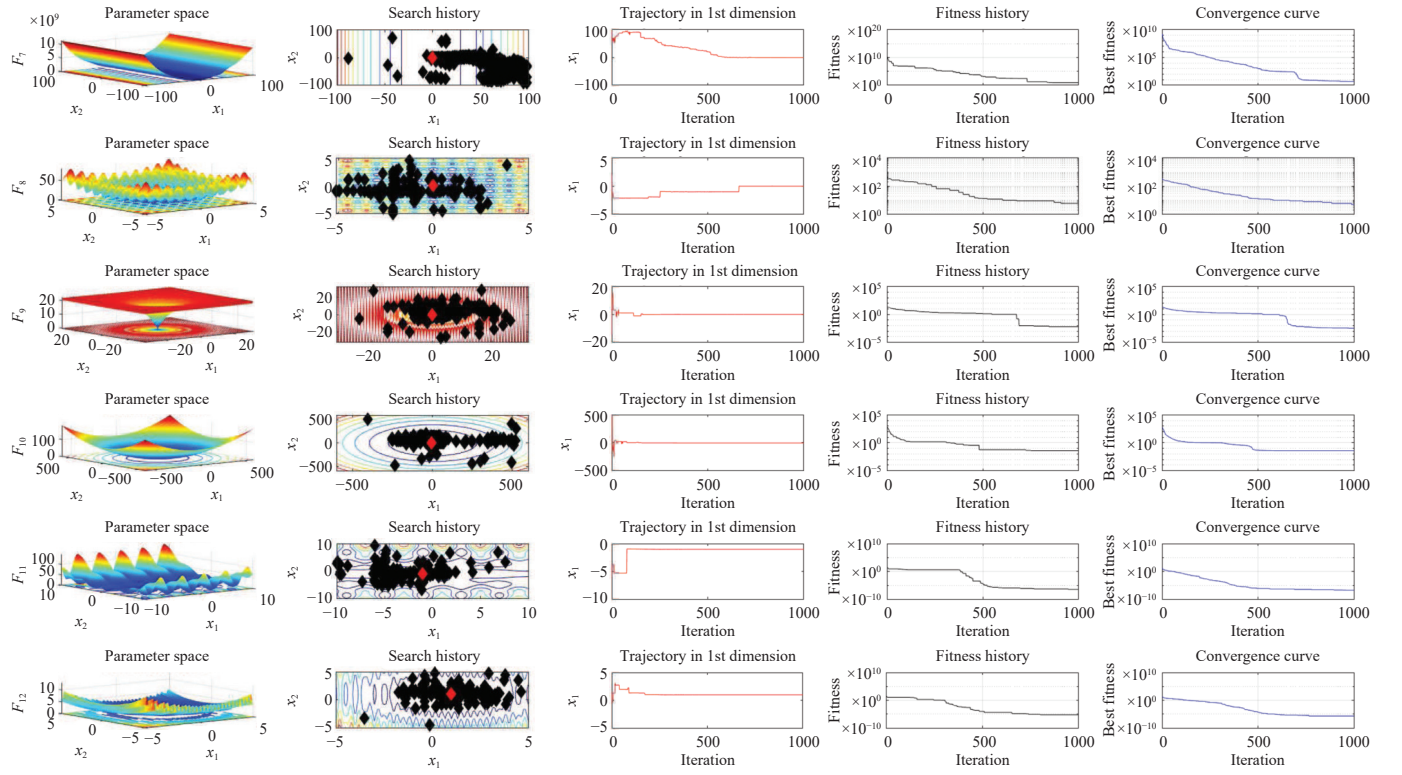


Fig. 2. 2D shapes, search histories, trajectories, fitness histories and convergence curves of F_7 – F_{12} with SBAGO.

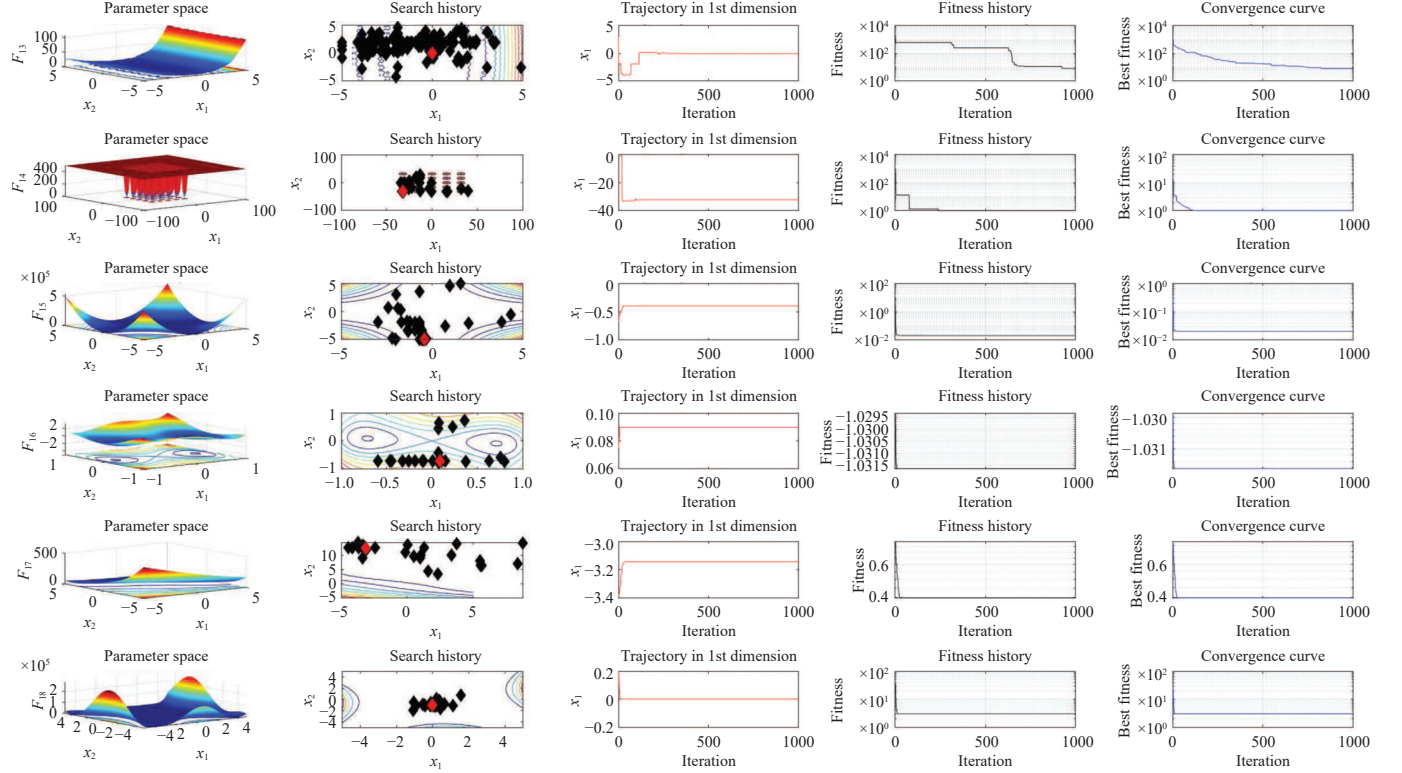


Fig. 3. 2D shapes, search histories, trajectories, fitness histories and convergence curves of F_{13} – F_{18} with SBAGO.

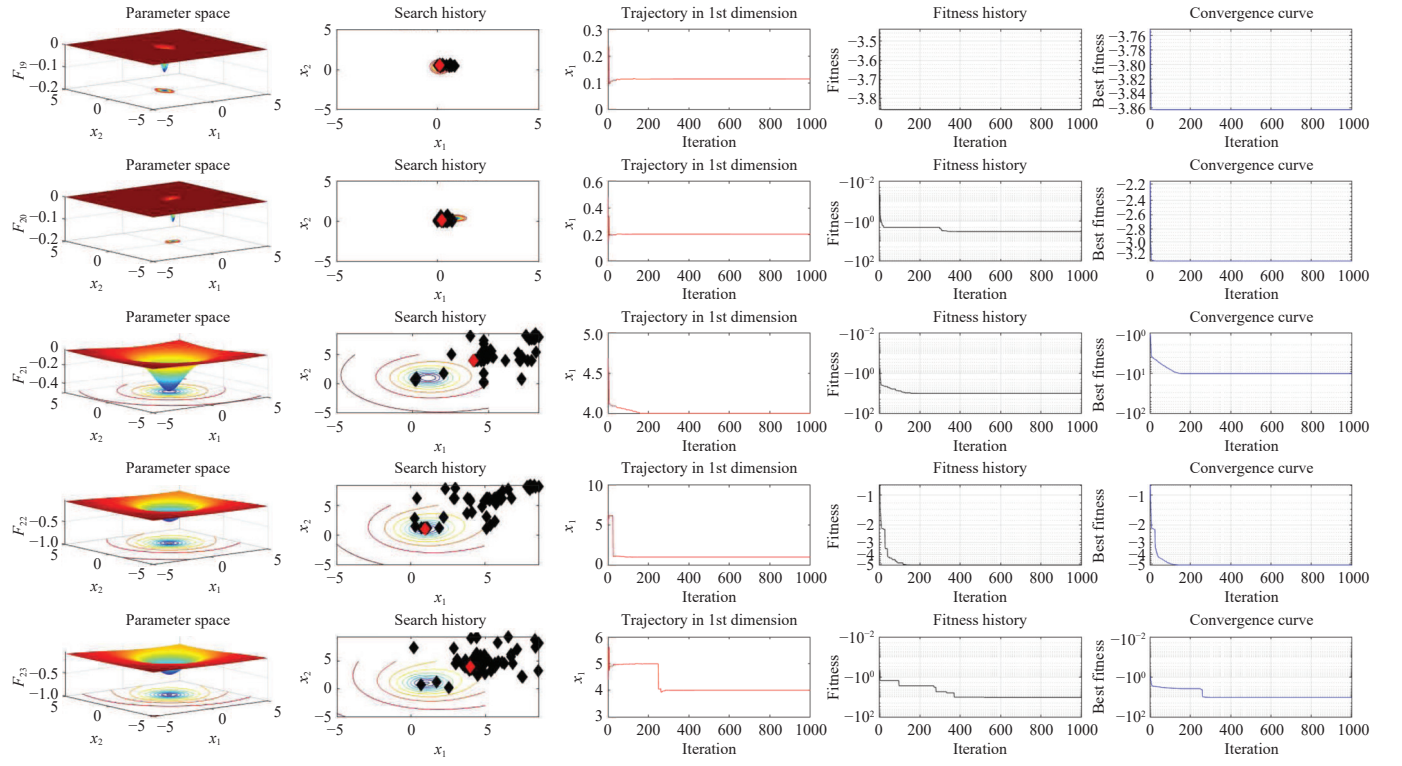


Fig. 4. 2D shapes, search histories, trajectories, fitness histories and convergence curves of F_{19} – F_{23} with SBAGO.

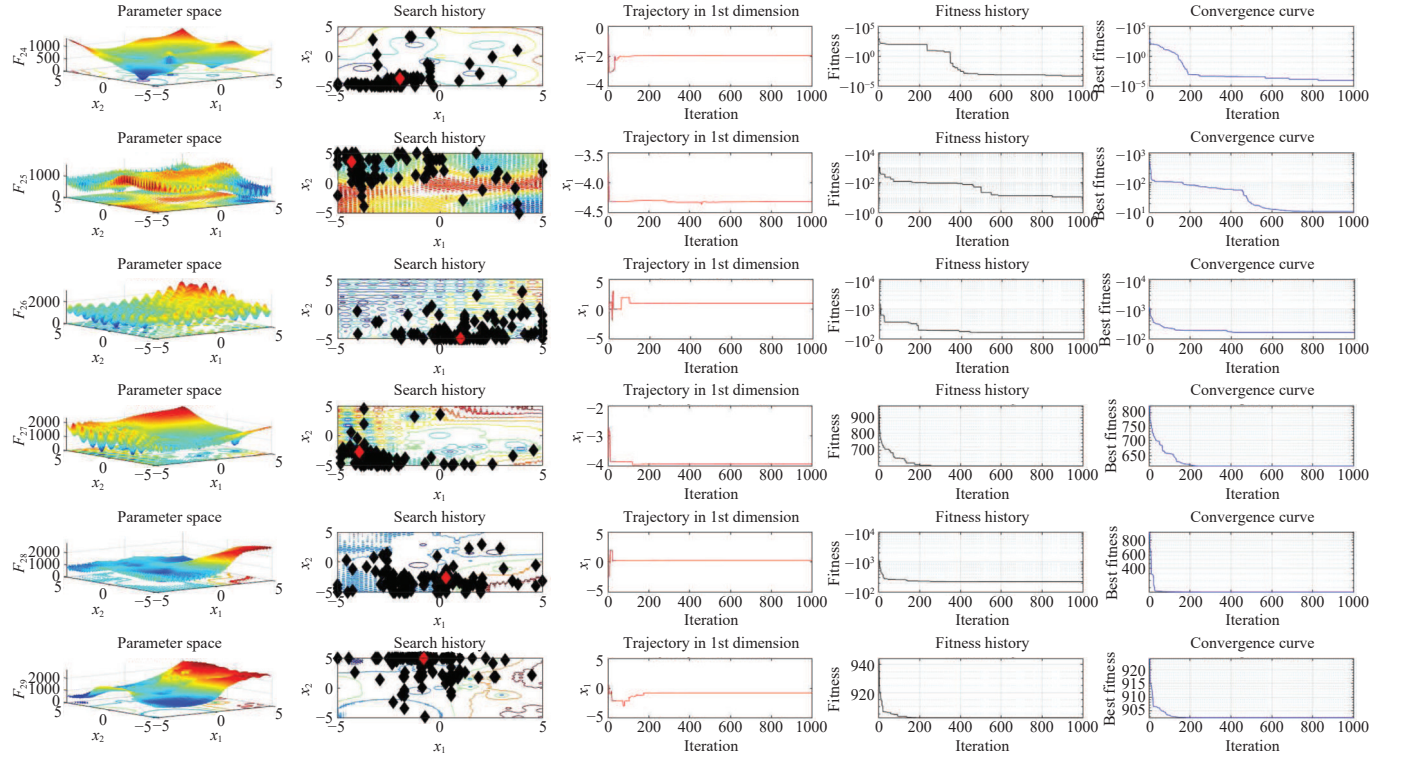


Fig. 5. 2D shapes, search histories, trajectories, fitness histories and convergence curves of F_{24} – F_{29} with SBAGO.