Supplementary File for "A Novel Multiobjective Fireworks Algorithm and Its Applications to Imbalanced Distance Minimization Problems"

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THE Supplementary File for "A Novel Multiobjective Fireworks Algorithm and Its Applications to Imbalanced Distance Minimization Problems" is given in this part to support the research in the main body. The detailed experimental results and analysis are presented in Section I, and the definition of imbalanced distance minimization problems is given in Section II.

I. Detailed Experimental Results and Analysis

Table S1 presents 22 multimodal multiobjective benchmark functions, where "MMF" is the function name, "PS geometry" and "PF geometry" are the shapes of PS and PF, "N_O" and "N_D" are the dimensions of objective and decision space.

Table S2 presents the results to explain the reason why use the smaller one as the final distance, where "+" indicates that IMOFWA*s* (using smaller one as the final distance) performs significantly better than IMOFWA*b* (using larger one as the final distance), "–" indicates otherwise, and "≈" means no significant difference between them. It indicates that using the smaller one can obtain better results in terms of P'_{SP} values.

TABLE S1
22 Benchmark Functions From CEC2019

MMF	N_O	N_D	PS geometry	PF geometry
MMF1	2	2	Nonlinear	Convex
MMF2	2	2	Nonlinear	Convex
MMF3	2	2	Nonlinear	Convex
MMF4	2	2	Nonlinear	Concave
MMF5	2	2	Nonlinear	Convex
MMF6	2	2	Nonlinear	Convex
MMF7	2	2	Nonlinear	Convex
MMF8	2	2	Nonlinear	Concave
MMF9	2	2	Linear	Convex
MMF10	2	2	Linear	Convex
MMF11	2	2	Linear	Convex
MMF12	2	2	Linear	Convex
MMF13	2	3	Nonlinear	Convex
MMF14	3	3	Linear	Concave
MMF15	3	3	Linear	Concave
MMF16	2	2	Nonlinear	Convex
MMF17	2	2	Nonlinear	Convex
MMF18	3	3	Nonlinear	Concave
MMF19	3	3	Nonlinear	Concave
MMF20	2	2	Linear	Convex
MMF21	2	2	Linear	Convex
MMF22	2	3	Linear	Convex

TABLE S2
Comparison Among IMFWA*s* and IMFWA*b*

MMF	IMFWA*s*	IMFWA*b*
MMF1	**3.86E−02**	3.91E−02 +
MMF2	**2.40E−02**	2.48E−02 ≈
MMF3	**1.94E−02**	2.17E−02 +
MMF4	2.31E−02	**2.30E−02** ≈
MMF5	**7.06E−02**	7.10E−02 ≈
MMF6	6.34E−02	**6.29E−02** ≈
MMF7	**2.22E−02**	2.24E−02 ≈
MMF8	**5.27E−02**	5.60E−02 +
MMF9	**6.49E−03**	6.62E−03 ≈
MMF10	1.64E−01	**1.64E−01** ≈
MMF11	**3.41E−01**	3.92E−01 +
MMF12	**2.82E−01**	3.67E−01 +
MMF13	**3.27E−01**	3.44E−01 ≈
MMF14	5.17E−02	**5.13E−02** ≈
MMF15	**1.38E−01**	1.48E−01 +
MMF16	2.83E−02	**2.83E−02** ≈
MMF17	**3.72E−01**	4.11E−01 +
MMF18	5.91E−02	**5.88E−02** ≈
MMF19	**1.57E−01**	1.62E−01 +
MMF20	**2.52E−01**	3.31E−01 +
MMF21	**3.46E−01**	3.82E−01 +
MMF22	**3.66E−01**	3.90E−01 +
AR	**1.2727**	1.7273
+/=/−	−/−/−	11/11/0

Table S3 presents the results of IMFWA, IMFWA$_D$ and IMFWA$_O$ in terms of P'_{SP}, and and the smallest P'_{SP} is marked bold. From it, IMFWA$_D$ has the smallest P'_{SP} on function MMF3−5, 14 and 18, IMFWA$_O$ performs the best on function MMF6, 9, 13 and 21. IMFWA has the best P'_{SP} on 13 MMFs, which is the most.

TABLE S3

MEAN P'_{SP} FROM IMFWA, IMFWA$_D$ AND IMFWA$_O$

MMF	IMFWA	IMFWA$_D$	IMFWA$_O$
MMF1	**3.86E−02**	3.93E−02 +	3.86E−02 ≈
MMF2	**2.40E−02**	2.57E−02 ≈	2.56E−02 ≈
MMF3	1.94E−02	**1.93E−02** ≈	2.21E−02 +
MMF4	2.31E−02	**2.31E−02** ≈	2.36E−02 ≈
MMF5	7.06E−02	**7.01E−02** ≈	7.06E−02 ≈
MMF6	6.34E−02	6.45E−02 +	**6.31E−02** ≈
MMF7	**2.22E−02**	2.29E−02 +	2.24E−02 ≈
MMF8	**5.27E−02**	5.38E−02 +	5.48E−02 +
MMF9	6.49E−03	6.47E−03 ≈	**6.45E−03** ≈
MMF10	**1.64E−01**	1.66E−01 ≈	1.67E−01 ≈
MMF11	**3.41E−01**	5.41E−01 +	4.72E−01 +
MMF12	**2.82E−01**	3.41E−01 +	3.06E−01 +
MMF13	3.27E−01	3.42E−01 ≈	**2.97E−01** ≈
MMF14	5.17E−02	**5.13E−02** ≈	5.14E−02 ≈
MMF15	**1.38E−01**	1.48E−01 +	1.47E−01 +
MMF16	**2.83E−02**	2.86E−02 ≈	2.86E−02 ≈
MMF17	**3.72E−01**	3.88E−01 +	4.12E−01 +
MMF18	5.91E−02	**5.87E−02** ≈	5.87E−02 ≈
MMF19	**1.57E−01**	1.59E−01 ≈	1.60E−01 +
MMF20	**2.52E−01**	3.51E−01 +	2.54E−01 ≈
MMF21	3.46E−01	3.49E−01 +	**2.49E−01** ≈
MMF22	**3.66E−01**	4.18E−01 +	3.83E−01 +
+/≈/−	−/−/−	11/11/0	8/14/0
AR	**1.5909**	2.3182	2.0909

TABLE S4

COMPARISON OF IMFWA, MFWA, MFWAa AND MFWAs IN TERMS OF P'_{SP}

MMF	IMFWA	MFWA	MFWAa	MFWAs
MMF1	**3.86E−02**	4.12E−02 +	3.90E−02 +	4.09E−02 +
MMF2	**2.40E−02**	2.95E−02 +	2.53E−02 ≈	2.76E−02 +
MMF3	**1.94E−02**	2.52E−02 +	1.99E−02 +	2.55E−02 +
MMF4	**2.31E−02**	2.48E−02 +	2.36E−02 ≈	2.48E−02 +
MMF5	**7.06E−02**	7.32E−02 ≈	7.13E−02 ≈	7.20E−02 ≈
MMF6	6.34E−02	6.32E−02 ≈	**6.29E−02** ≈	6.31E−02 ≈
MMF7	**2.22E−02**	2.38E−02 +	2.24E−02 ≈	2.34E−02 +
MMF8	**5.27E−02**	6.96E−02 +	5.47E−02 +	7.14E−02 +
MMF9	**6.49E−03**	7.67E−03 +	6.59E−03 ≈	7.39E−03 +
MMF10	**1.64E−01**	1.67E−01 ≈	1.68E−01 ≈	1.86E−01 +
MMF11	3.41E−01	6.25E−01 ≈	**3.10E−01** ≈	4.41E−01 ≈
MMF12	**2.82E−01**	4.21E−01 ≈	3.44E−01 +	5.03E−01 ≈
MMF13	3.27E−01	3.89E−01 ≈	**3.14E−01** ≈	3.17E−01 ≈
MMF14	5.17E−02	5.22E−02 ≈	**5.12E−02** ≈	5.20E−02 ≈
MMF15	**1.38E−01**	1.49E−01 +	1.40E−01 ≈	1.42E−01 ≈
MMF16	**2.83E−02**	3.16E−02 +	2.88E−02 +	3.08E−02 +
MMF17	**3.72E−01**	4.79E−01 +	4.25E−01 +	4.08E−01 ≈
MMF18	5.91E−02	5.90E−02 ≈	5.91E−02 ≈	**5.85E−02** ≈
MMF19	1.57E−01	1.66E−01 +	**1.56E−01** ≈	1.60E−01 ≈
MMF20	**2.52E−01**	7.30E−01 +	2.72E−01 +	7.08E−01 +
MMF21	3.46E−01	1.60E+00 +	**3.39E−01** ≈	1.57E+00 +
MMF22	**3.66E−01**	5.64E−01 +	4.39E−01 +	5.18E−01 +
+/≈/−	−/−/−	14/8/0	8/14/0	12/10/0
AR	**1.5000**	3.6364	1.9091	2.9545

Table S4 presents the results of IMFWA, MFWA, MFWAa and MFWAs in terms of P'_{SP}, and the smallest P'_{SP} is marked bold. From it, MFWA performs poorly on all MMFs, and MFWAs only performs well on function MMF18. The results of MFWAa are the best on functions MMF6, 11, 13, 14, 19 and 21. IMFWA has the best P'_{SP} on MMF1−5, MMF7−10, 12, 15–17, 20 and 22. Thus it is clear that IMFWA is the best among them.

Table S5 presents the results of IMFWA, IMO_Ring_PSO_SCD and IMODE in terms of P'_{SP}, which shows that the proposed adaptive strategy and special archive guidance can help IMFWA more.

The results of P'_{SP}, I_{GDX} and H'_V of IMFWA, Omni_optimizer, DN-NSGAII, MO_Ring_PSO_SCD, SMMOPIO, SMPSO_MM, MODE_CSCD, SSMOPSO and MMOEAC are shown in Tables S6–S8, respectively. In them, the average rankings (AR) obtained by Friedman test are presented at their bottom, and the smallest mean P'_{SP}, I_{GDX}, H'_V and the best average ranking are marked in bold. From Table S6, MODE_CSCD has the best P'_{SP} on MMF3, 8, 11, 15 and 19–21. SMPSO_MM performs the best on MMF7 and 9. IMFWA has better P'_{SP} than other algorithms on 10 MMFs. According to AR, IMFWA takes the 1st place, and MODE_CSCD and SMPSO_MM take the 2nd and 3rd places, respectively. Omni_optimizer and DN-NSGAII are the worst among them in terms of AR. The results of pair-wise Wilcoxon rank sum tests also indicate that IMFWA is significantly better than its eight peers in terms of P'_{SP}. From Table S7, it is easy to find that the I_{GDX} results obtained by nine methods are similar to their P'_{SP} results. The proposed algorithm outperforms its peers in terms of I_{GDX}. For H'_V results shown in Table S8, the number of the best H'_V value obtained by IMFWA, Omni_optimizer, SMMOPIO, DN-NSGAII, SMPSO_MM, MODE_CSCD, SSMOPSO and MMOEAC is 6, 7, 2, 5, 1, 1, 0 and 0, respectively. IMFWA provides better results in 6, 12, 17, 5, 6, 6, 4 and 7 cases, equivalent in 10, 3, 3, 13, 12, 15, 17 and 15 cases when comparing it with Omni_optimizer, DN-NSGAII, MO_Ring_PSO_SCD, SMMOPIO, SMPSO_MM, MODE_CSCD, SSMOPSO and MMOEAC. In the pair-wise comparisons, the H'_V results achieved by IMFWA are superior or competitive to its peers'. In the conclusion, IMFWA has better performance than its peers on most MMFs in terms of P'_{SP}, I_{GDX} and H'_V values.

Tables S9 and S10 present the mean P'_{SP} and I_{GDX} values of IMFWA and the top 5 optimizers participating in the CEC2019 competition including NMOHSA, PEN_MOBA, CEOA, MMO-ClusteringPSO and MM-NAEMO. The best mean P'_{SP} and I_{GDX} on each function are highlighted. The average rankings (AR) obtained by Friedman test are pre-

TABLE S5
COMPARISON OF IMFWA, IMO_RING_PSO_SCD AND IMODE IN TERMS OF P'_{SP}

MMF	IMFWA	IMO_Ring_PSO_SCD	IMODE
MMF1	**3.86E−02**	4.88E−02 +	4.73E−02 +
MMF2	2.40E−02	3.08E−02 +	**1.81E−02** −
MMF3	1.94E−02	2.60E−02 +	**1.76E−02** −
MMF4	**2.31E−02**	2.67E−02 +	2.91E−02 +
MMF5	**7.06E−02**	8.51E−02 +	7.97E−02 +
MMF6	**6.34E−02**	7.25E−02 +	6.77E−02 +
MMF7	**2.22E−02**	2.71E−02 +	2.73E−02 +
MMF8	**5.27E−02**	6.84E−02 +	7.71E−02 +
MMF9	**6.49E−03**	8.00E−03 +	6.61E−03 ≈
MMF10	**1.64E−01**	1.77E−01 +	8.86E−01 ≈
MMF11	**3.41E−01**	5.72E−01 ≈	8.74E−01 ≈
MMF12	**2.82E−01**	5.00E−01 ≈	1.29E+00 +
MMF13	**3.27E−01**	3.34E−01 ≈	5.38E−01 +
MMF14	**5.17E−02**	5.34E−02 +	5.58E−02 +
MMF15	**1.38E−01**	1.47E−01 +	1.93E−01 +
MMF16	**2.83E−02**	3.55E−02 +	3.63E−02 +
MMF17	**3.72E−01**	4.86E−01 +	7.70E−01 +
MMF18	**5.91E−02**	6.09E−02 +	6.56E−02 +
MMF19	**1.57E−01**	1.61E−01 ≈	1.88E−01 +
MMF20	2.52E−01	2.96E−01 +	**7.53E−02** −
MMF21	3.46E−01	3.87E−01 +	**8.70E−02** −
MMF22	3.66E−01	5.00E−01 +	**9.43E−02** −
AR	**1.2273**	2.4091	2.3636
+/=/−	−/−/−	18/4/0	14/3/5

sented at the bottom of tables, and the best AR is in bold. From Table S9, it can be seen that IMFWA is more stable than its peers due to its AR. MMO-ClusteringPSO achieves the best P'_{SP} values on MMF1, 4−7, 9, 14, 16 and 18. COEA has the best P'_{SP} values on MMF10−13, 20−22. MM-NAEMO is the best on MMF2, 3, 15 and 19. NMOHSA only perform the best on MMF17. According to the AR, IMFWA is the best among them. Moreover, the results of pair-wise Wilcoxon rank sum tests, except COEA and MMO-ClusteringPSO due to their incomplete data, show that IMFWA provides better results in all 10 cases, and equivalent results in 6, 7 and 3 cases in comparison with NMOHSA, PEN_MOBA and MM-NAEMO, respectively. From Table S10, it can be observed that IMFWA and NMOHSA perform the best on only one function. MMO-ClusteringPSO achieves the best I_{GDX} values on nine functions, which is the most. COEA has the best I_{GDX} values on seven functions, which is the second. MM-NAEMO is the best on four functions. However, IMFWA is the best among them with the AR of 3.1818. Moreover, comparing with NMOHSA, PEN_MOBA and MM-NAEMO, the results of pair-wise tests show that IMFWA provides better results in 9, 10 and 10 cases, comparable results in 7, 8 and 3 cases, respectively. In conclusion, the results of mean P'_{SP} and I_{GDX} values indicate that the proposed algorithm has better performance than top 5 participating algorithms benchmarked on CEC2019 benchmark functions in terms of AR in decision space.

Figs. S1 and S2 show that the visualization results of PSs

TABLE S6
MEAN P'_{SP} VALUES OF IMFWA, OMNI_OPTIMIZER, DN-NSGAII, MO_RING_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO AND MMOEAC

MMF	IMFWA	Omni_optimizer	DN-NSGAII	MO_Ring_PSO_SCD	SMMOPIO	SMPSO_MM	MMODE_CSCD	SSMOPSO	MMOEAC
MMF1	**1.04E−02**	2.02E−02 +	2.42E−02 +	1.49E−02 +	1.26E−02 +	1.17E−02 +	1.31E−02 +	1.22E−02 +	1.20E−02 +
MMF2	6.39E−03	2.10E−02 +	1.69E−02 +	9.50E−03 +	4.13E−03 −	6.77E−03 ≈	4.30E−03 −	7.85E−03 +	**2.63E−03** −
MMF3	5.00E−03	1.71E−02 +	1.64E−02 +	7.64E−03 +	3.19E−03 −	5.21E−03 +	**2.93E−03** −	5.58E−03 +	2.49E−02 +
MMF4	**6.76E−03**	2.66E−02 +	2.47E−02 +	8.63E−03 +	8.61E−03 +	7.30E−03 +	6.82E−03 +	7.29E−03 +	9.89E−03 +
MMF5	**2.24E−02**	6.80E−02 +	7.02E−02 +	3.01E−02 +	3.26E−02 +	2.49E−02 +	2.59E−02 +	2.49E−02 +	2.70E−02 +
MMF6	**2.23E−02**	5.47E−02 +	5.78E−02 +	2.75E−02 +	2.81E−02 +	2.37E−02 +	2.32E−02 +	2.44E−02 +	2.56E−02 +
MMF7	6.92E−03	9.73E−03 +	1.20E−02 +	9.10E−03 +	7.14E−03 +	**6.89E−03** ≈	6.99E−03 +	8.35E−03 +	9.81E−03 +
MMF8	**1.56E−02**	7.41E−02 +	6.02E−02 +	2.15E−02 +	1.70E−02 +	1.60E−02 +	**1.56E−02** ≈	1.64E−02 +	1.76E−02 +
MMF9	2.08E−03	6.43E−02 +	5.91E−02 +	2.76E−03 +	2.06E−03 ≈	**1.82E−03** +	2.37E−03 +	2.25E−03 +	2.53E−03 +
MMF10	**1.61E−01**	8.26E+00 +	8.23E+00 +	1.67E−01 +	5.09E+00 +	2.80E+00 −	1.69E−01 +	1.62E−01 +	7.58E+00 +
MMF11	1.87E−01	2.47E+00 +	2.27E+00 +	7.59E−01 +	2.11E+00 +	2.04E+00 ≈	**1.59E−01** −	2.24E−01 +	2.62E+00 +
MMF12	**1.43E−01**	3.22E+00 +	3.14E+00 +	6.20E−01 +	2.77E+00 +	1.82E+00 +	7.17E−01 +	1.55E+00 +	9.55E−01 +
MMF13	2.47E−01	5.67E−01 +	5.67E−01 +	3.31E−01 +	5.25E−01 +	5.08E−01 +	2.15E−01 −	2.54E−01 +	**2.03E−01** −
MMF14	**3.14E−02**	6.04E−02 +	6.87E−02 +	3.32E−02 +	3.85E−02 +	3.35E−02 +	3.15E−02 +	4.08E−02 +	3.43E−02 +
MMF15	1.19E−01	4.46E−01 +	4.25E−01 +	1.45E−01 +	1.66E−01 +	1.47E−01 +	**2.56E−02** −	1.35E−01 +	3.56E−02 −
MMF16	**7.38E−03**	2.18E−02 +	2.28E−02 +	1.09E−02 +	9.17E−03 +	8.44E−03 +	8.59E−03 +	9.17E−03 +	8.64E−03 +
MMF17	1.29E−01	1.14E+00 +	1.02E+00 +	2.23E−01 +	2.25E−01 +	1.65E−01 +	1.50E−01 +	1.74E−01 +	**1.24E−01** −
MMF18	**3.49E−02**	7.57E−02 +	8.77E−02 +	3.76E−02 +	4.36E−02 +	3.76E−02 +	3.82E−02 +	3.69E−02 +	4.67E−02 +
MMF19	1.25E−01	2.54E−01 +	2.55E−01 +	1.59E−01 +	1.75E−01 +	1.55E−01 +	**2.85E−02** −	1.59E−01 +	6.90E−02 −
MMF20	7.56E−02	2.75E+00 +	2.81E+00 +	4.70E−02 −	1.83E−02 −	3.15E−02 −	**1.63E−02** −	1.91E−02 −	2.58E−02 −
MMF21	7.71E−02	1.21E+00 +	1.34E+00 +	5.44E−02 −	2.28E−02 −	6.01E−02 ≈	**1.57E−02** −	2.51E−02 −	3.41E−02 −
MMF22	1.23E−01	1.10E+00 +	9.82E−01 +	1.34E−01 ≈	5.67E−02 −	1.19E−01 ≈	7.40E−02 −	8.05E−02 −	**3.73E−02** −
+/≈/−	−/−/−	22/0/0	22/0/0	19/1/2	16/1/5	15/5/2	12/1/9	19/0/3	14/0/8
AR	**2.5227**	8.3864	8.3864	5.4545	4.9773	3.9545	2.6736	4.1136	4.5909

TABLE S7

MEAN I_{GDX} VALUES OF IMFWA, OMNI_OPTIMIZER, DN-NSGAII, MO_RING_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO AND MMOEAC

MMF	IMFWA	Omni_optimizer	DN-NSGAII	MO_Ring_PSO_SCD	SMMOPIO	SMPSO_MM	MMODE_CSCD	SSMOPSO	MMOEAC
MMF1	**1.04E−02**	2.01E−02	2.41E−02	1.49E−02	1.26E−02	1.17E−02	1.30E−02	1.22E−02	1.20E−02
MMF2	6.31E−03	2.02E−02	1.67E−02	9.35E−03	4.07E−03	6.69E−03	4.30E−03	7.69E−03	**2.64E−03**
MMF3	4.89E−03	1.70E−02	1.60E−02	7.49E−03	3.15E−03	5.11E−03	**2.93E−03**	5.49E−03	2.49E−02
MMF4	**6.75E−03**	2.66E−02	2.47E−02	8.61E−03	8.60E−03	7.28E−03	6.82E−03	7.27E−03	9.89E−03
MMF5	**2.24E−02**	6.78E−02	7.01E−02	3.00E−02	3.25E−02	2.49E−02	2.59E−02	2.49E−02	2.70E−02
MMF6	**2.22E−02**	5.45E−02	5.77E−02	2.75E−02	2.81E−02	2.37E−02	2.31E−02	2.43E−02	2.56E−02
MMF7	6.92E−03	9.69E−03	1.20E−02	9.08E−03	7.13E−03	**6.88E−03**	6.95E−03	8.35E−03	9.80E−03
MMF8	**1.56E−02**	7.31E−02	5.98E−02	2.14E−02	1.69E−02	1.60E−02	**1.56E−02**	1.63E−02	1.76E−02
MMF9	2.08E−03	6.43E−02	5.91E−02	2.76E−03	2.06E−03	**1.82E−03**	2.29E−03	2.25E−03	2.53E−03
MMF10	**1.61E−01**	2.00E−01	2.00E−01	1.65E−01	1.90E−01	1.77E−01	1.67E−01	1.62E−01	2.00E−01
MMF11	1.87E−01	2.50E−01	2.50E−01	2.21E−01	2.49E−01	2.49E−01	**1.58E−01**	2.24E−01	2.49E−01
MMF12	**1.43E−01**	2.46E−01	2.47E−01	1.84E−01	2.46E−01	2.35E−01	2.12E−01	2.43E−01	2.39E−01
MMF13	2.07E−01	2.68E−01	2.68E−01	2.26E−01	2.51E−01	2.47E−01	2.14E−01	2.05E−01	**1.44E−01**
MMF14	**3.14E−02**	6.04E−02	6.87E−02	3.32E−02	3.85E−02	3.35E−02	3.15E−02	4.08E−02	3.43E−02
MMF15	1.19E−01	2.52E−01	2.52E−01	1.45E−01	1.66E−01	1.47E−01	**2.56E−02**	1.35E−01	3.56E−02
MMF16	**7.38E−03**	2.16E−02	2.27E−02	1.09E−02	9.15E−03	8.43E−03	8.57E−03	9.16E−03	8.65E−03
MMF17	1.29E−01	8.09E−01	7.13E−01	2.10E−01	2.16E−01	1.61E−01	1.48E−01	1.67E−01	**1.28E−01**
MMF18	**3.49E−02**	7.57E−02	8.77E−02	3.76E−02	4.35E−02	3.76E−02	3.82E−02	3.69E−02	4.67E−02
MMF19	1.24E−01	2.12E−01	2.16E−01	1.56E−01	1.69E−01	1.54E−01	**2.84E−02**	1.59E−01	7.84E−02
MMF20	7.52E−02	2.75E+00	2.81E+00	4.70E−02	1.83E−02	3.15E−02	**1.63E−02**	1.91E−02	2.59E−02
MMF21	7.64E−02	1.13E+00	1.26E+00	5.42E−02	2.28E−02	5.98E−02	**1.57E−02**	2.51E−02	3.41E−02
MMF22	1.21E−01	1.08E+00	9.73E−01	1.33E−01	5.65E−02	1.18E−01	7.40E−02	8.02E−02	**3.73E−02**
+/≈/−	−/−/−	22/0/0	22/0/0	19/1/2	15/1/6	14/4/4	13/1/8	18/0/4	14/0/8
AR	**2.5227**	8.2727	8.5227	5.3864	4.9773	3.9091	2.7945	4.1591	4.5455

TABLE S8

MEAN H'_{V} VALUES OF IMFWA, OMNI_OPTIMIZER, DN-NSGAII, MO_RING_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO AND MMOEAC

MMF	IMFWA	Omni_optimizer	DN-NSGAII	MO_Ring_PSO_SCD	SMMOPIO	SMPSO_MM	MMODE_CSCD	SSMOPSO	MMOEAC
MMF1	**1.14**	1.14 +	1.14 +	1.14 +	1.14 ≈	1.14 ≈	1.14 ≈	1.14 ≈	1.14 ≈
MMF2	1.14	**1.14** ≈	1.14 +	1.15 +	1.14 ≈	1.15 +	1.14 +	1.15 +	1.15 +
MMF3	**1.14**	1.14 +	1.14 +	1.15 +	1.14 ≈	1.15 +	1.14 +	1.15 +	1.15 +
MMF4	1.84	**1.84** −	1.84 +	1.85 +	1.84 ≈	1.84 ≈	1.84 ≈	1.84 ≈	1.85 +
MMF5	**1.14**	1.14 +	1.14 +	1.14 +	1.14 +	1.14 ≈	1.14 ≈	1.14 ≈	1.14 ≈
MMF6	1.14	1.14 ≈	1.14 +	1.14 +	**1.14** −	1.14 ≈	1.14 ≈	1.14 ≈	1.14 ≈
MMF7	1.14	1.14 ≈	1.14 +	1.14 +	1.14 ≈	1.14 ≈	**1.14** ≈	1.14 ≈	1.14 ≈
MMF8	2.36	**2.36** −	2.36 ≈	2.37 +	2.36 ≈	2.36 −	2.36 ≈	2.36 ≈	2.36 ≈
MMF9	0.10	0.10 ≈	**0.10** −	0.10 +	0.10 ≈	0.10 ≈	0.10 ≈	0.10 ≈	0.10 ≈
MMF10	0.08	0.08 ≈	**0.08** −	0.08 +	0.08 ≈	0.08 ≈	0.08 ≈	0.08 ≈	0.08 ≈
MMF11	0.07	0.07 ≈	0.07 +	0.07 +	0.07 ≈	**0.07** ≈	0.07 ≈	0.07 ≈	0.07 ≈
MMF12	0.64	0.64 +	**0.64** −	0.64 ≈	0.64 ≈	0.64 ≈	0.64 ≈	0.64 ≈	0.64 ≈
MMF13	0.05	0.05 ≈	**0.05** −	0.05 +	0.05 ≈	0.05 ≈	0.05 ≈	0.05 ≈	0.05 ≈
MMF14	**0.31**	0.33 +	0.32 +	0.34 +	0.34 +	0.33 +	0.35 +	0.34 +	0.35 +
MMF15	**0.22**	0.23 +	0.23 +	0.23 +	0.23 +	0.23 +	0.25 +	0.22 ≈	0.26 +
MMF16	1.14	1.14 ≈	1.14 +	1.14 +	**1.14** ≈	1.14 ≈	1.14 ≈	1.14 ≈	1.14 ≈
MMF17	1.15	**1.14** −	1.16 ≈	1.15 ≈	1.14 −	1.15 −	1.15 ≈	1.15 ≈	1.15 ≈
MMF18	**0.28**	0.33 +	0.32 +	0.34 +	0.32 +	0.33 +	0.35 +	0.29 +	0.35 +
MMF19	0.23	0.23 −	**0.22** −	0.23 +	0.23 +	0.23 +	0.24 +	0.22 −	0.26 +
MMF20	0.06	**0.06** −	0.06 −	0.06 −	0.06 −	0.06 −	0.06 ≈	0.06 ≈	0.06 ≈
MMF21	0.06	**0.06** −	0.06 −	0.06 −	0.06 −	0.06 −	0.06 −	0.06 ≈	0.06 ≈
MMF22	0.02	**0.02** ≈	0.02 ≈	0.02 −	0.02 −	0.02 −	0.02 ≈	0.02 ≈	0.02 ≈
+/≈/−	−/−/−	6/10/6	12/3/7	17/3/2	5/13/4	6/12/4	6/15/1	4/17/1	7/15/0

TABLE S9

MEAN P'_{SP} VALUES OF IMFWA AND TOP FIVE COMPETITORS

MMF	IMFWA	NMOHSA	PEN_MOBA	CEOA	MMO-ClusteringPSO	MM-NAEMO
MMF1	3.86E−02	4.30E−02 +	4.07E−02 +	4.52E−02	**3.28E−02**	4.87E−02 +
MMF2	2.40E−02	2.32E−02 −	3.07E−02 +	2.12E−02	5.18E−02	**1.18E−02** −
MMF3	1.94E−02	2.12E−02 ≈	2.34E−02 +	3.53E−02	2.86E−02	**1.39E−02** −
MMF4	2.31E−02	2.49E−02 +	2.41E−02 +	2.53E−02	**1.29E−02**	3.12E−02 +
MMF5	7.06E−02	8.07E−02 +	7.43E−02 +	7.88E−02	**5.66E−02**	8.72E−02 +
MMF6	6.34E−02	7.09E−02 +	6.62E−02 ≈	6.81E−02	**4.41E−02**	7.43E−02 +
MMF7	2.22E−02	2.17E−02 ≈	2.14E−02 −	2.77E−02	**1.27E−02**	2.29E−02 +
MMF8	5.27E−02	7.46E−02 +	**5.19E−02** ≈	7.24E−02	5.22E−02	3.95E−01 +
MMF9	6.49E−03	5.83E−03 −	6.65E−03 ≈	6.65E−03	**4.08E−03**	5.07E−03 ≈
MMF10	1.64E−01	1.61E−01 ≈	5.72E−01 ≈	**9.48E−03**	1.66E−01	1.24E−02 −
MMF11	3.41E−01	8.30E−01 +	4.18E−01 ≈	**7.42E−03**	6.06E−01	4.19E−02 −
MMF12	2.82E−01	5.11E−01 ≈	4.11E−01 +	**3.19E−03**	7.25E−01	5.02E−03 −
MMF13	3.27E−01	3.76E−01 +	3.28E−01 −	**1.07E−01**	3.64E−01	2.72E−01 ≈
MMF14	5.17E−02	5.42E−02 +	5.49E−02 +	5.18E−02	**2.65E−02**	4.66E−02 ≈
MMF15	1.38E−01	1.42E−01 ≈	1.40E−01 ≈	5.78E−02	1.65E−01	**5.28E−02** −
MMF16	2.83E−02	3.11E−02 +	3.04E−02 +	3.21E−02	**2.28E−02**	3.46E−02 +
MMF17	3.72E−01	**3.64E−01** −	8.08E−01 +	6.45E−01	6.53E−01	4.54E−01 +
MMF18	5.91E−02	6.26E−02 +	6.25E−02 +	7.98E−02	**3.13E−02**	6.62E−02 +
MMF19	1.57E−01	1.63E−01 ≈	1.58E−01 ≈	9.29E−02	1.60E−01	**8.70E−02** −
MMF20	2.52E−01	9.61E−02 −	8.13E−02 −	**5.34E−02**	1.29E−01	1.14E−01 −
MMF21	3.46E−01	1.17E−01 −	1.60E−01 −	**7.93E−02**	2.44E−01	9.72E−01 +
MMF22	3.66E−01	1.16E−01 −	1.77E−01 −	**1.03E−01**	3.57E−01	1.46E−01 −
+/≈/−	−/−/−	10/6/6	10/7/5	-/-/-	-/-/-	10/3/9
AR	**3.1818**	3.9545	3.7727	3.2273	3.3182	3.5454

TABLE S10

MEAN I_{GDX} VALUES OF IMFWA AND TOP FIVE COMPETITORS

MMF	IMFWA	NMOHSA	PEN_MOBA	CEOA	MMO-ClusteringPSO	MM-NAEMO
MMF1	3.85E−02	4.27E−02 +	4.05E−02 +	4.50E−02	**3.26E−02**	4.87E−02 +
MMF2	2.31E−02	2.21E−02 −	2.97E−02 +	2.11E−02	4.75E−02	**1.18E−02** −
MMF3	1.86E−02	1.99E−02 ≈	2.27E−02 +	3.50E−02	2.70E−02	**1.39E−02** −
MMF4	2.30E−02	2.46E−02 +	2.39E−02 +	2.52E−02	**1.29E−02**	3.12E−02 +
MMF5	7.03E−02	8.02E−02 +	7.39E−02 +	7.85E−02	**5.63E−02**	8.71E−02 +
MMF6	6.31E−02	7.04E−02 +	6.57E−02 ≈	6.78E−02	**4.39E−02**	7.42E−02 +
MMF7	2.21E−02	2.15E−02 ≈	2.12E−02 −	2.76E−02	**1.26E−02**	2.28E−02 +
MMF8	**5.04E−02**	7.30E−02 +	5.14E−02 ≈	7.13E−02	5.18E−02	3.53E−01 +
MMF9	6.48E−03	5.83E−03 −	6.64E−03 ≈	6.65E−03	**4.07E−03**	5.05E−03 ≈
MMF10	1.61E−01	1.60E−01 ≈	1.71E−01 ≈	**9.48E−03**	1.61E−01	1.23E−02 −
MMF11	2.04E−01	2.32E−01 +	2.06E−01 ≈	**7.42E−03**	2.18E−01	4.18E−02 −
MMF12	1.69E−01	1.76E−01 ≈	1.75E−01 +	**3.19E−03**	1.96E−01	5.02E−03 −
MMF13	2.31E−01	2.30E−01 ≈	2.29E−01 ≈	**9.99E−02**	2.34E−01	1.88E−01 ≈
MMF14	5.17E−02	5.42E−02 +	5.49E−02 +	5.17E−02	**2.65E−02**	4.66E−02 ≈
MMF15	1.38E−01	1.42E−01 ≈	1.40E−01 ≈	5.78E−02	1.65E−01	**5.28E−02** −
MMF16	2.82E−02	3.09E−02 +	3.02E−02 +	3.20E−02	**2.26E−02**	3.46E−02 +
MMF17	3.55E−01	**3.43E−01** −	6.51E−01 +	5.18E−01	5.43E−01	4.26E−01 +
MMF18	5.90E−02	6.25E−02 +	6.23E−02 +	7.98E−02	**3.13E−02**	6.62E−02 +
MMF19	1.56E−01	1.61E−01 ≈	1.57E−01 ≈	9.29E−02	1.56E−01	**8.65E−02** −
MMF20	2.51E−01	9.60E−02 −	8.12E−02 −	**5.34E−02**	1.29E−01	1.13E−01 −
MMF21	3.40E−01	1.17E−01 −	1.59E−01 −	**7.89E−02**	2.41E−01	8.20E−01 +
MMF22	3.62E−01	1.15E−01 −	1.76E−01 −	**1.03E−01**	3.54E−01	1.46E−01 −
+/≈/−	−/−/−	9/7/6	10/8/4	−/−/−	−/−/−	10/3/9
AR	**3.1818**	3.8636	3.7727	3.2727	3.3636	3.5455

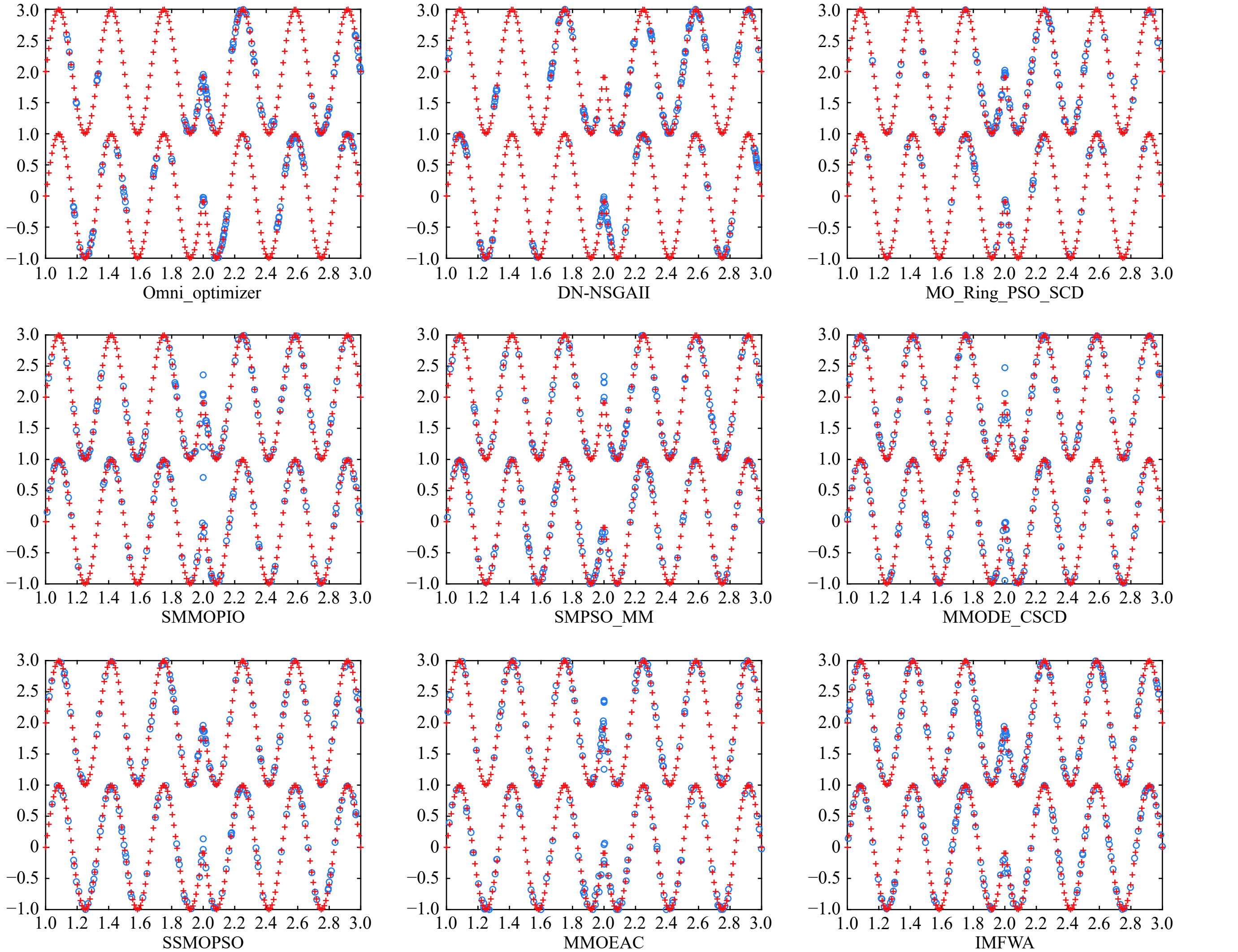
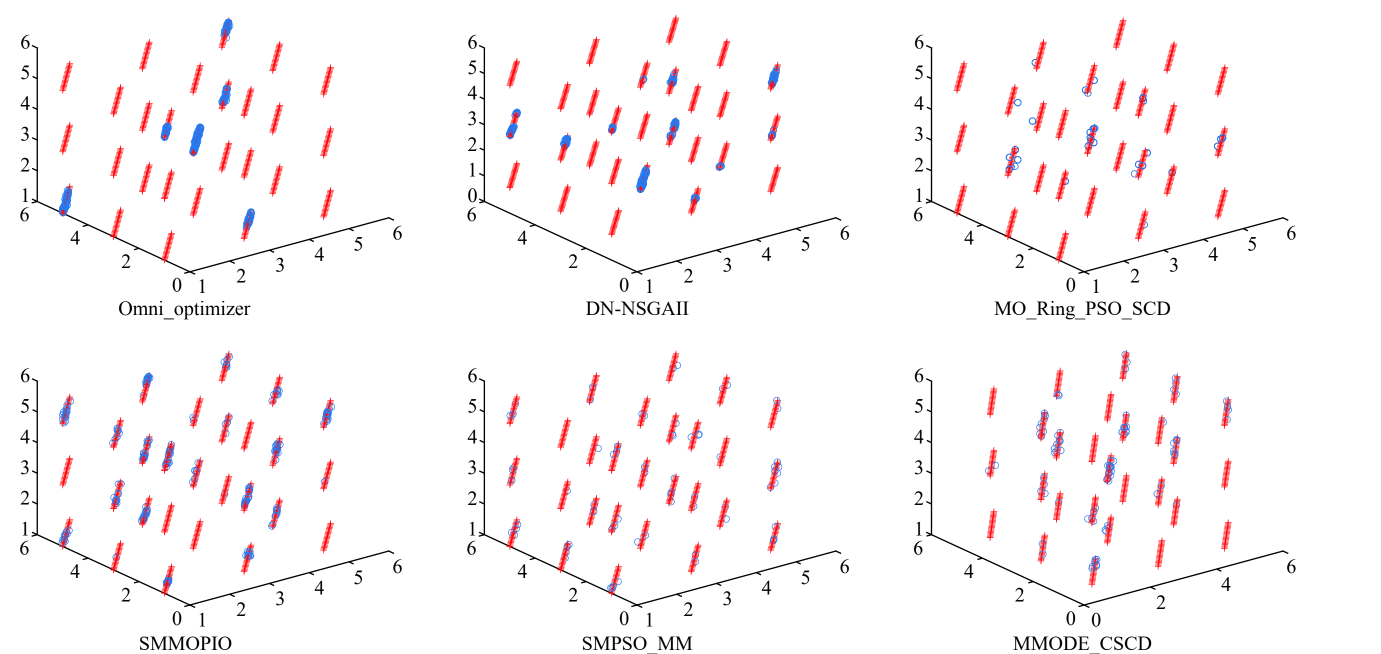


Fig. S1. The PSs on MMF5 are obtained by Omni_optimizer, DN-NSGAII, MO_Ring_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO, MMOEAC and IMFWA, where red "+" means true PSs, and blue circle means obtained PSs.



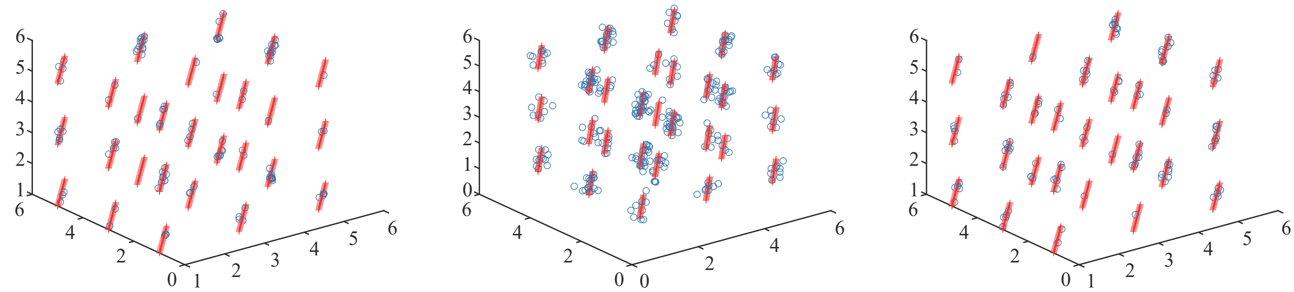


Fig. S2. The PSs on MMF22 are obtained by Omni_optimizer, DN-NSGAII, MO_Ring_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO, MMOEAC and IMFWA, where red "+" means true PSs, and blue circle means obtained PSs.

obtained by Omni_optimizer, DN-NSGAII, MO_Ring_PSO_SCD, SMMOPIO, SMPSO_MM, MMODE_CSCD, SSMOPSO, MMOEAC and IMFWA on MMF5 and MMF22, respectively.

Fig. S3 presents the process of switching between explosion and random strategies on MMF4 from CEC2019. It can be seen that our algorithm tends to select explosion strategy to converge the true PS quickly in the early stage, while a random one is selected more frequently in the later stage aiming to locate more equivalent true PS.

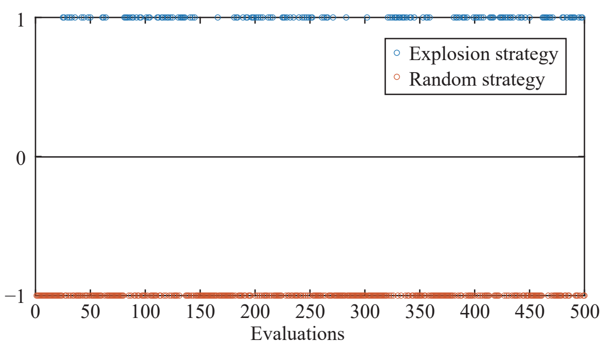


Fig. S3. The process of switching between explosion and random strategies on MMF4 from CEC2019.

II. Imbalanced Distance Minimization Problems (IDMPs)

We first give the definition of imbalanced distance minimization problems, it is:

Definition 1: An MMOP is defined to be an IDMP, if it satisfies one or both of the following conditions:

1) For a point on the PF, solutions close to one equivalent Pareto optimal solution are more likely to dominate solutions close to another equivalent Pareto optimal solution.

2) For a point on the PF, the complexity of searching for one equivalent Pareto optimal solution is lower than that of another equivalent Pareto optimal solution.

Note that complexity can be seen as the required computation cost, i.e., the more evaluations needed to find an equivalent solution, the higher the complexity. It can be found that the existing MMEAs prefer to select the convergent solutions rather than diversity ones since they use a convergence-first selection criterion, and thus the solutions in them are overly crowded around an equivalent Pareto optimal subset, while other equivalent Pareto optimal subsets are rather sparsely located. This work designs an adaptive strategy to preserve high solution diversity in decision space.