# Supplementary Materials for "A Reference Vector-Guided and Generative Adversarial Network-Driven Competitive Swarm Optimizer for Large-Scale Multi-Objective Optimization"

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***Abstract*—This is the supplementary material for the paper "*A Reference Vector-Guided and Generative Adversarial Network-Driven Competitive Swarm Optimizer for Large-Scale Multi-Objective Optimization*". This file includes the extra information not included in the main paper. These contents are used to support the results and discussions in the main paper.**

## A. Computational Complexity Analysis of RG-CSO

As presented in Algorithm 1 of the main paper, the main loop of RG-CSO consists of four essential procedures: swarm division, loser particles update, winner particles update, and environmental selection.

1) The dual-criteria competition procedure requires a time complexity of $O(mN^2)$.
2) The reference vector-guided learning strategy involves two computationally intensive operations: matching each loser with its appropriate winner, which requires $O(mN^2)$, and updating the velocities and positions of loser particles, which takes $O(mnN)$.
3) For the GAN model training described in Algorithm S1, executing $E_p$ epochs demands $O(E_p \sum_{l=1}^{d} s_l s_{l-1} n^2 N^2)$, where $l$ indexes the network layer, $d$ represents the number of layers, $s_l$ indicates the number of neurons in the $l$th layer, and $s_{l-1}$ denotes the number of input channels for the $l$th layer. The training frequency in the GAN-driven reproduction strategy is determined by the parameter $t_r$.
4) The computational complexity of calculating the APD value and selecting elite solutions is $O(mN^2)$.

Overall, the worst-case computational complexity of RG-CSO within one generation is $\max\{O(mN^2), O(mnN), O(E_p/t_r \sum_{l=1}^{d} s_l s_{l-1} n^2 N^2)\}$, demonstrating its dependence on several key factors: the number of decision variables, the number of objective functions, the swarm size, the GAN model's neural network architecture, as well as GAN training epochs and training frequency.

## B. GAN Model Training

The pseudocode of the GAN training process in RG-CSO, as described in Section III-D of the main paper, is presented in Algorithm S1. The trained GAN model and generated dataset $Z$ are then used in the designed GAN-driven reproduction strategy to generate new particles.

**Algorithm S1:** GAN Model Training

**Input:** Loser group $L$ and winner group $W$
**Output:** Trained GAN model and generated dataset $Z$

Compute mean vector $\mu$ and covariance matrix $\Sigma$ of $W$;
**while** *training epochs not completed* **do**
  Shuffle $W$ and use them as real samples;
  $Z \leftarrow$ Sample $N/2$ data points from $\mathcal{N}(\mu, \Sigma)$;
  $G(Z) \leftarrow$ Produce samples by inputting $Z$ into $G$;
  Combine $G(Z)$ and $L$ as fake samples;
  $D \leftarrow$ Update the discriminator with Eq.(8) in the main paper;
  $Z \leftarrow$ Sample $N/2$ new data points from $\mathcal{N}(\mu, \Sigma)$;
  $G(Z) \leftarrow$ Produce samples by inputting $Z$ into $G$;
  $G \leftarrow$ Update the generator with Eq.(9) in the main paper;
**end**

## C. Characteristics of Benchmark Problems

The characteristics of LSMOP1-LSMOP9 benchmark problems adopted in the experiments are presented in Table S1.

TABLE S1
Characteristics of benchmark problems.

| Problems | Pareto front | Modality | Separability |
|---|---|---|---|
| LSMOP1 | Linear | Unimodal | Fully Separable |
| LSMOP2 | Linear | Mixed | Partially Separable |
| LSMOP3 | Linear | Multimodal | Mixed |
| LSMOP4 | Linear | Mixed | Mixed |
| LSMOP5 | Concave | Unimodal | Fully Separable |
| LSMOP6 | Concave | Mixed | Partially Separable |
| LSMOP7 | Concave | Multimodal | Mixed |
| LSMOP8 | Concave | Mixed | Mixed |
| LSMOP9 | Disconnected | Mixed | Fully Separable |

## D. Supportive Experimental Results

The following tables (Tables S2-S15) and figures (Figs. S1-S3) provide supportive experimental results for the discussions in Section-IV of the main paper.

TABLE S2

IGD RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON TWO-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 5.7772e-3 (2.88e-4) − | 3.9518e-3 (1.35e-4) − | 2.9050e-1 (3.74e-2) − | 1.7898e-3 (2.01e-6) − | 1.8083e-3 (4.30e-6) − | 1.0586e-2 (1.82e-3) − | 3.9145e-3 (1.81e-4) − | 1.4788e-3 (2.41e-3) |
| | 200 | 2.4027e-2 (1.07e-3) − | 6.9024e-3 (7.28e-4) + | 3.4372e-1 (2.04e-2) − | 1.8846e-3 (1.52e-5) = | 2.0938e-3 (8.40e-5) + | 1.2137e-1 (1.16e-2) − | 4.5791e-3 (2.24e-4) − | 2.1645e-3 (8.92e-3) |
| | 500 | 2.2041e-1 (5.31e-3) − | 6.5974e-2 (2.24e-2) − | 3.8582e-1 (1.50e-2) − | 4.0627e-3 (8.09e-4) + | 1.4046e-1 (6.44e-2) − | 2.0881e-1 (7.40e-3) − | 1.7338e-2 (2.28e-3) − | 3.2746e-3 (3.44e-3) |
| | 1000 | 6.8378e-1 (6.46e-4) − | 2.2238e-1 (1.86e-2) − | 4.0795e-1 (6.78e-3) − | 2.5542e-1 (3.53e-2) − | 2.8440e-1 (3.65e-3) − | 2.7975e-1 (1.02e-2) − | 1.6903e-1 (3.07e-2) − | 1.0343e-1 (5.74e-2) |
| LSMOP2 | 100 | 1.8898e-1 (1.70e-3) − | 7.4462e-3 (5.89e-4) + | 8.3437e-2 (3.85e-3) − | 6.6774e-3 (2.42e-3) + | 7.6012e-3 (5.75e-3) + | 2.4175e-2 (4.38e-3) − | 5.4249e-3 (7.97e-4) + | 1.1936e-2 (1.02e-3) |
| | 200 | 1.4126e-1 (7.35e-4) − | 1.3993e-2 (8.98e-4) = | 5.4535e-2 (1.20e-3) − | 3.6134e-2 (1.37e-2) − | 2.7954e-2 (1.52e-2) − | 1.9926e-2 (9.41e-4) − | 7.7733e-3 (1.52e-3) + | 1.3515e-2 (9.81e-4) |
| | 500 | 6.3751e-2 (2.80e-4) − | 1.1797e-2 (2.99e-4) + | 2.6490e-2 (8.05e-4) − | 3.1410e-2 (6.70e-4) − | 3.1560e-2 (8.45e-4) − | 1.1579e-2 (3.44e-4) + | 1.5447e-2 (7.39e-4) = | 1.3334e-2 (1.35e-3) |
| | 1000 | 3.2821e-2 (7.83e-5) − | 8.9879e-3 (2.03e-4) + | 1.5815e-2 (5.71e-4) − | 2.0391e-2 (2.87e-4) − | 2.0351e-2 (2.17e-4) − | 6.8994e-3 (1.89e-4) + | 9.1934e-3 (3.82e-4) = | 9.2002e-3 (6.91e-4) |
| LSMOP3 | 100 | 8.7796e-1 (1.84e-1) − | 1.1204e+0 (2.62e-1) − | 1.5186e+0 (2.67e-3) − | 7.0708e-1 (6.38e-5) − | 6.9379e-1 (4.23e-2) − | 7.0550e-1 (8.81e-3) − | 7.8130e-1 (2.88e-1) − | 5.4461e-1 (1.41e-1) |
| | 200 | 1.7958e+0 (4.97e-1) − | 1.4140e+0 (2.96e-2) − | 1.5568e+0 (1.06e-3) − | 7.0711e-1 (1.44e-6) − | 1.6306e+0 (7.78e-1) − | 8.6139e-1 (1.43e-1) − | 1.1977e+0 (5.77e-1) − | 6.9742e-1 (5.12e-1) |
| | 500 | 4.2716e+0 (4.96e-1) − | 1.5442e+0 (1.07e-2) − | 1.5787e+0 (7.51e-5) − | 7.0714e-1 (6.55e-6) − | 8.4715e+0 (1.95e+0) − | 1.3753e+0 (2.07e-1) − | 2.1093e+0 (6.51e-1) − | 6.8327e-1 (3.72e-1) |
| | 1000 | 2.2214e+0 (8.21e-2) − | 1.5657e+0 (2.42e-3) = | 1.5857e+0 (1.69e-7) = | 7.0727e-1 (2.37e-5) = | 1.5122e+1 (2.16e+0) − | 2.8715e+0 (4.70e+0) − | 2.4888e+0 (6.83e-1) − | 1.6005e+0 (1.33e+0) |
| LSMOP4 | 100 | 4.8007e-2 (8.99e-3) − | 7.5564e-3 (1.61e-3) − | 2.0116e-1 (7.41e-3) − | 3.7173e-3 (1.11e-3) − | 1.2122e-2 (1.63e-3) − | 2.1663e-2 (4.30e-3) − | 4.7829e-3 (1.05e-3) − | 2.0783e-3 (1.92e-3) |
| | 200 | 5.5150e-2 (6.93e-3) − | 1.0339e-2 (4.90e-4) − | 1.5012e-1 (6.17e-3) − | 9.8149e-3 (1.09e-3) − | 1.0937e-2 (1.07e-3) − | 1.8393e-2 (3.35e-3) − | 1.0284e-2 (6.55e-4) − | 4.1684e-3 (1.41e-3) |
| | 500 | 4.5668e-2 (3.00e-4) − | 1.5501e-2 (1.82e-3) − | 9.3702e-2 (4.37e-3) − | 1.2222e-2 (1.36e-3) − | 1.4061e-2 (1.25e-3) − | 3.2867e-2 (2.12e-3) − | 9.7615e-3 (2.54e-4) = | 9.7189e-3 (9.40e-3) |
| | 1000 | 1.6981e-2 (1.39e-4) + | 2.0425e-2 (7.94e-4) + | 5.8631e-2 (2.79e-3) − | 3.5575e-2 (1.50e-3) − | 3.2771e-2 (1.11e-3) = | 2.1792e-2 (4.97e-4) + | 1.7232e-2 (9.54e-4) + | 3.1356e-2 (5.99e-3) |
| LSMOP5 | 100 | 1.2179e-2 (1.17e-3)− | 3.7018e-3 (1.76e-4) − | 7.4209e-1 (3.39e-16) − | 2.0266e-3 (1.69e-5) = | 2.1516e-3 (4.64e-5) + | 1.3959e-2 (3.36e-3) − | 3.1675e-3 (9.79e-5) − | 2.1927e-3 (8.71e-2) |
| | 200 | 6.0654e-2 (5.02e-3)− | 2.0248e-2 (6.20e-3) − | 7.4209e-1 (3.39e-16) − | 2.3508e-3 (1.03e-4) − | 5.2081e-2 (1.88e-1) − | 6.1762e-2 (1.64e-2) − | 5.3518e-2 (1.87e-1) − | 1.6712e-3 (8.44e-2) |
| | 500 | 5.9574e-1 (9.80e-3)− | 7.1414e-1 (4.69e-2) − | 7.4209e-1 (3.39e-16) − | 6.0810e-3 (5.69e-4) − | 3.0392e-1 (1.43e-1) − | 7.3218e-1 (2.69e-2) − | 7.7072e-2 (1.26e-1) − | 5.8753e-3 (5.48e-2) |
| | 1000 | 1.9556e-1 (2.70e-3) − | 7.4209e-1 (3.39e-16) − | 7.4209e-1 (3.39e-16) − | 2.0845e-1 (1.13e-1) − | 4.3100e-1 (6.25e-2) − | 7.4209e-1 (3.39e-16) − | 3.1219e-1 (1.07e-1) − | 1.3765e-1 (4.69e-2) |
| LSMOP6 | 100 | 8.7737e-1 (4.68e-2) − | 3.0511e-1 (1.30e-1) − | 6.5475e-1 (5.19e-3) − | 7.4242e-1 (8.34e-5) − | 7.1677e-1 (8.09e-2) − | 6.0292e-1 (1.51e-1) − | 6.0831e-1 (2.62e-1) − | 2.8828e-1 (2.81e-1) |
| | 200 | 7.2957e-1 (4.12e-2) − | 3.6203e-1 (2.93e-2) + | 6.7112e-1 (2.22e-3) − | 5.5933e-1 (1.61e-1) − | 6.7880e-1 (1.09e-1) − | 6.7247e-1 (1.37e-1) − | 8.0203e-1 (1.90e-1) − | 4.0609e-1 (2.93e-1) |
| | 500 | 6.1790e+0 (6.11e+0) − | 3.5567e-1 (6.90e-3) + | 6.7259e-1 (6.39e-4) + | 7.4345e-1 (1.90e-3) − | 7.2604e-1 (1.00e-1) − | 6.0481e-1 (2.36e-1) + | 9.2176e-1 (5.29e-1) − | 6.5340e-1 (3.15e-1) |
| | 1000 | 3.1311e+0 (2.93e+0) = | 3.3848e-1 (1.32e-2) + | 6.7275e-1 (3.15e-4) + | 7.4220e-1 (4.28e-2) + | 7.5754e-1 (1.02e-2) + | 6.8668e-1 (1.31e-1) + | 1.3624e+0 (7.94e-1) = | 1.4859e+0 (9.04e-4) |
| LSMOP7 | 100 | 6.1015e+0 (1.48e+0) − | 8.7807e-1 (1.74e-1) − | 3.9574e+0 (3.02e+0) − | 1.1868e+0 (3.18e-1) − | 9.1439e-1 (3.16e-1) = | 1.1380e+0 (2.04e-1) − | 8.4314e-1 (1.10e-1) = | 9.9757e-1 (2.94e-1) |
| | 200 | 8.1323e+0 (2.49e+0) − | 1.4577e+0 (3.96e-3) − | 7.2784e+0 (4.42e+0) − | 1.1945e+0 (4.46e-1) − | 1.2303e+0 (5.33e-1) − | 1.4902e+0 (4.37e-3) − | 1.4559e+0 (5.32e-1) − | 1.0550e+0 (4.61e-1) |
| | 500 | 1.4321e+2 (1.18e+1) − | 1.4977e+0 (9.74e-4) − | 9.9719e+0 (6.04e+0) − | 1.0886e+0 (4.73e-1) = | 2.7189e+0 (3.16e+0) − | 1.5105e+0 (9.65e-4) − | 1.9733e+0 (4.09e-1) − | 1.0665e+0 (4.05e-1) |
| | 1000 | 3.9283e+1 (1.76e+0) − | 1.5083e+0 (5.78e-4) − | 1.3046e+1 (5.42e+0) − | 1.5177e+0 (2.73e-1) − | 1.4273e+5 (7.81e+5) − | 1.5161e+0 (1.48e-3) − | 2.2452e+0 (7.65e-1) − | 1.4682e+0 (4.26e-1) |
| LSMOP8 | 100 | 4.4981e-2 (2.19e-3) = | 1.3704e-2 (6.71e-3) = | 7.4209e-1 (3.39e-16) − | 8.8637e-3 (1.96e-3) + | 1.4274e-2 (4.33e-3) = | 3.4217e-2 (1.04e-2) = | 9.3484e-3 (1.00e-2) + | 6.9414e-2 (8.08e-2) |
| | 200 | 6.1240e-2 (2.82e-3) = | 3.7261e-2 (1.11e-2) + | 7.4209e-1 (3.39e-16) − | 1.5301e-2 (4.41e-3) + | 3.4949e-2 (4.79e-3) + | 6.6096e-2 (9.94e-3) = | 2.0212e-2 (1.05e-2) + | 1.0325e-1 (8.78e-2) |
| | 500 | 3.9010e-1 (1.10e-2) − | 7.2402e-1 (9.90e-2) − | 7.4279e-1 (3.83e-3) − | 2.5617e-2 (6.79e-4) + | 2.4409e-2 (5.56e-4) + | 2.0057e-1 (1.71e-2) − | 7.7479e-2 (1.33e-1) + | 1.8098e-1 (9.51e-2) |
| | 1000 | 1.1800e-1 (1.49e-3) + | 7.4209e-1 (3.39e-16) − | 7.4209e-1 (3.39e-16) − | 3.3347e-2 (1.98e-3) + | 3.1672e-2 (1.21e-3) + | 6.2252e-1 (4.46e-2) − | 9.0651e-2 (1.24e-1) + | 1.8742e-1 (8.16e-2) |
| LSMOP9 | 100 | 1.3277e-1 (2.51e-2) + | 8.1004e-1 (0.00e+0) − | 9.7621e-1 (1.19e-1) − | 3.0048e-1 (3.09e-1) − | 8.9104e-1 (6.96e-2) − | 4.2817e-1 (3.77e-1) − | 8.1004e-1 (6.24e-16) − | 2.0859e-1 (3.84e-3) |
| | 200 | 1.4617e-1 (7.97e-3) = | 8.1004e-1 (0.00e+0) − | 9.5798e-1 (4.20e-2) − | 3.2439e-1 (2.01e-1) − | 8.7066e-1 (5.22e-2) − | 6.0259e-1 (1.72e-1) − | 4.8385e-1 (1.11e-1) − | 2.9463e-1 (5.95e-2) |
| | 500 | 6.8591e-1 (3.51e-2) − | 5.2106e-1 (5.50e-2) − | 8.9649e-1 (9.18e-2) − | 4.6080e-1 (9.24e-3) − | 8.1975e-1 (3.95e-2) − | 7.4163e-1 (1.26e-1) − | 4.6781e-1 (1.12e-2) − | 4.0744e-1 (7.76e-3) |
| | 1000 | 1.9654e-1 (1.16e-2) + | 5.3207e-1 (1.29e-2) − | 9.9418e-1 (4.47e-2) − | 9.9904e-2 (1.59e-3) + | 2.2270e+0 (6.17e-1) − | 7.7980e-1 (8.91e-2) = | 5.0133e-1 (6.03e-2) + | 8.0549e-1 (5.55e-3) |
| +/−/= | | 4/28/4 | 9/24/3 | 2/33/1 | 8/24/4 | 7/26/3 | 5/28/3 | 8/23/5 | −− |
| *Best*/*All* | | 3/36 | 3/36 | 0/36 | 6/36 | 2/36 | 2/36 | 3/36 | 17/36 |

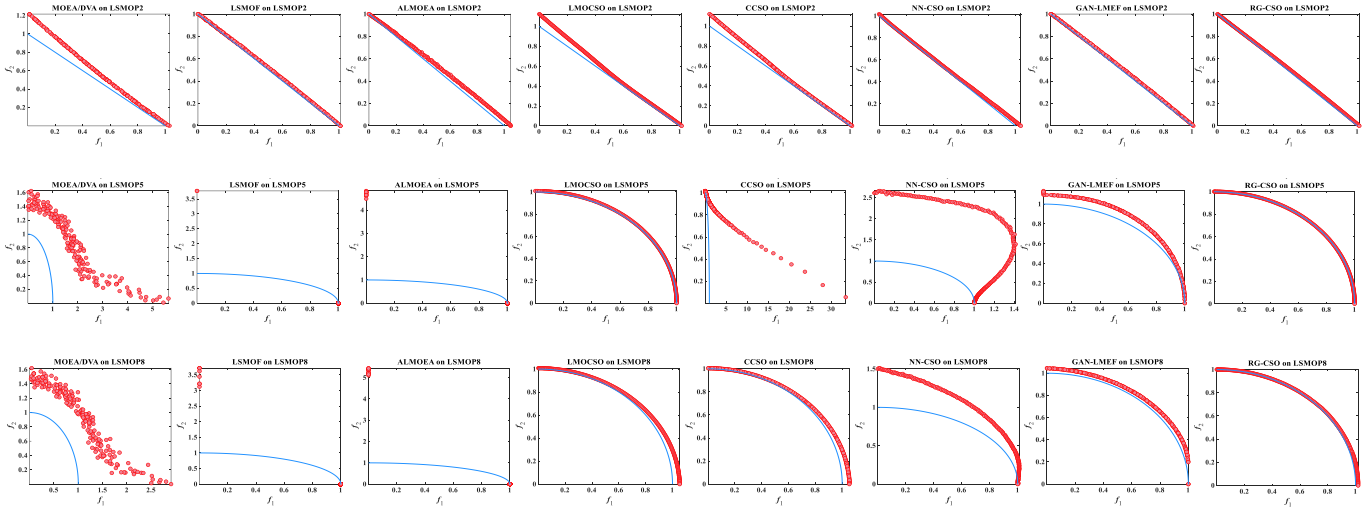



Fig. S1. Final solution sets obtained by RG-CSO and compared algorithms in solving LSMOP2, 5, 8 with $m = 2$, $n = 500$. The plots are based on the medium IGD over 30 independent runs. The true PF is represented by the blue curve, and the obtained solutions are shown by red scatter points.

TABLE S3

IGD RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 3.3193e-2 (9.48e-4) − | 2.1322e-1 (1.09e-3) − | 4.0810e-1 (6.58e-3) − | 2.3024e-2 (3.46e-4) = | 2.5995e-2 (3.14e-3) = | 1.5820e-1 (5.27e-2) − | 1.2834e-1 (3.96e-2) − | 2.3022e-2 (2.43e-3) |
| | 200 | 8.5209e-2 (1.46e-3) − | 2.1211e-1 (1.55e-3) − | 4.4672e-1 (6.28e-3) − | 4.4830e-2 (9.35e-3) = | 9.6966e-2 (1.81e-2) − | 2.9999e-1 (1.12e-2) − | 2.5122e-1 (1.90e-2) − | 4.0357e-2 (9.34e-3) |
| | 500 | 4.2689e-1 (6.79e-3) − | 2.1565e-1 (9.73e-4) − | 4.7818e-1 (6.48e-3) − | 2.2721e-1 (1.70e-2) − | 2.0897e-1 (8.08e-3) − | 3.6051e-1 (1.53e-2) − | 4.8306e-1 (2.92e-2) − | 1.5276e-1 (1.99e-2) |
| | 1000 | 1.4247e-1 (1.03e-3) + | 2.8796e-1 (2.14e-2) = | 4.8839e-1 (4.15e-3) − | 5.8139e-1 (2.91e-2) − | 4.0512e-1 (1.40e-2) − | 3.9342e-1 (3.57e-2) − | 5.7077e-1 (5.61e-3) − | 2.7130e-1 (1.65e-2) |
| LSMOP2 | 100 | 1.2840e-1 (1.21e-3) − | 1.4433e-1 (5.86e-3) − | 1.3256e-1 (4.52e-3) − | 4.3736e-2 (1.09e-3) − | 6.1393e-2 (3.37e-3) − | 7.0800e-2 (3.64e-3) − | 1.9506e-1 (5.30e-3) − | 3.7170e-2 (2.46e-3) |
| | 200 | 1.0411e-1 (7.62e-4) − | 1.2695e-1 (1.97e-3) − | 8.9827e-2 (2.35e-3) − | 5.5223e-2 (8.43e-4) − | 6.1142e-2 (1.01e-3) − | 4.8292e-2 (1.86e-3) − | 1.1651e-1 (1.71e-3) − | 4.0547e-2 (2.61e-3) |
| | 500 | 5.6844e-2 (6.45e-4) − | 6.8790e-2 (1.64e-3) − | 5.2972e-2 (6.75e-4) − | 3.7830e-2 (2.02e-4) − | 3.8667e-2 (3.17e-4) − | 3.6085e-2 (7.41e-4) − | 6.1109e-2 (8.50e-4) − | 3.5923e-2 (2.58e-3) |
| | 1000 | 3.8463e-2 (7.47e-4) − | 4.8459e-2 (1.20e-3) − | 3.9401e-2 (1.47e-4) − | 3.0150e-2 (1.43e-4) − | 2.9355e-2 (1.61e-4) + | 2.6149e-2 (2.23e-4) + | 4.4498e-2 (9.26e-4) − | 3.0085e-2 (1.30e-3) |
| LSMOP3 | 100 | 7.6293e-1 (8.74e-2) − | 4.1005e-1 (1.42e-1) − | 8.6070e-1 (4.19e-5) − | 3.6657e-1 (2.64e-2) − | 7.4119e-1 (7.81e-2) − | 8.0031e-1 (4.55e-2) − | 6.5247e-1 (1.46e-1) = | 3.5606e-1 (1.97e-1) |
| | 200 | 1.6203e+0 (2.94e-1) − | 5.2235e-1 (1.61e-1) = | 8.6071e-1 (1.40e-5) − | 5.2840e-1 (2.87e-2) = | 2.3266e+0 (1.55e+0) − | 8.4315e-1 (3.14e-2) − | 8.3398e-1 (3.52e-2) = | 5.3767e-1 (1.74e-1) |
| | 500 | 5.4192e+0 (1.32e+0) − | 7.3531e-1 (4.04e-2) = | 8.6071e-1 (1.18e-5) − | 1.1745e+0 (4.22e-1) − | 5.5173e+0 (4.42e-1) − | 8.5478e-1 (2.06e-2) − | 8.6070e-1 (4.66e-5) − | 7.3012e-1 (1.18e-1) |
| | 1000 | 2.2847e+0 (1.10e-1) − | 7.6274e-1 (3.80e-2) − | 8.6012e-1 (2.02e-3) = | 4.4100e+0 (4.89e-1) − | 7.4019e+0 (5.82e-1) − | 8.4187e-1 (3.48e-2) − | 8.6072e-1 (1.16e-16) − | 7.3286e-1 (1.23e-1) |
| LSMOP4 | 100 | 1.2260e-1 (1.65e-2) − | 2.2938e-1 (2.87e-2) − | 3.7682e-1 (1.52e-2) − | 5.7616e-2 (1.49e-2) − | 9.7420e-2 (2.04e-2) = | 1.6241e-1 (1.47e-2) − | 2.1873e-1 (7.52e-3) − | 5.1799e-2 (2.39e-2) |
| | 200 | 1.2804e-1 (4.87e-3) = | 2.1886e-1 (3.83e-3) − | 2.4303e-1 (3.84e-3) − | 1.0984e-1 (1.25e-2) + | 1.2047e-1 (9.22e-3) = | 1.5625e-1 (6.42e-3) − | 2.7054e-1 (7.79e-3) − | 1.2186e-1 (1.04e-2) |
| | 500 | 9.0628e-2 (6.22e-4) = | 1.7665e-1 (3.51e-3) − | 1.4235e-1 (3.03e-3) − | 1.2148e-1 (2.72e-3) − | 1.1507e-1 (2.72e-3) − | 8.7956e-2 (3.06e-3) = | 1.8084e-1 (2.55e-3) − | 9.1169e-2 (4.41e-3) |
| | 1000 | 4.6367e-2 (7.59e-4) + | 1.2508e-1 (3.22e-3) − | 9.0932e-2 (2.03e-3) − | 7.9545e-2 (3.22e-4) − | 7.9595e-2 (3.25e-4) − | 5.3512e-2 (1.37e-3) − | 1.0917e-1 (1.25e-3) − | 5.2065e-2 (5.90e-3) |
| LSMOP5 | 100 | 5.2181e-2 (1.11e-3) − | 3.5889e-1 (2.21e-3) − | 5.2415e-1 (1.44e-2) − | 2.8969e-2 (5.85e-5) − | 1.8154e-1 (9.09e-2) − | 2.5576e-1 (2.14e-2) − | 8.8740e-2 (3.84e-2) − | 2.7435e-2 (8.64e-3) |
| | 200 | 2.6624e-1 (1.40e-2) − | 3.6989e-1 (1.70e-2) − | 5.3929e-1 (3.06e-3) − | 3.2524e-2 (1.43e-3) + | 4.4382e-1 (5.55e-2) − | 3.0558e-1 (8.06e-3) − | 3.4728e-1 (1.01e-2) − | 1.0731e-1 (1.07e-1) |
| | 500 | 9.1496e-1 (2.10e-2) − | 3.9444e-1 (2.89e-2) − | 5.4082e-1 (1.23e-5) − | 4.7411e-1 (1.12e-2) − | 5.2818e-1 (6.72e-3) − | 4.4029e-1 (7.83e-3) − | 3.9918e-1 (3.54e-2) − | 1.5572e-1 (9.60e-2) |
| | 1000 | 4.0952e-1 (3.10e-3) − | 5.0265e-1 (3.03e-2) − | 5.4083e-1 (1.15e-5) − | 9.9667e-1 (8.60e-2) − | 7.6722e-1 (2.57e-2) − | 4.9302e-1 (1.01e-2) − | 4.2045e-1 (1.17e-2) − | 2.5281e-1 (5.95e-2) |
| LSMOP6 | 100 | 2.4799e+0 (9.18e-1) − | 6.7323e-1 (1.80e-1) + | 1.1563e+0 (6.80e-2) − | 7.3686e-1 (8.23e-2) = | 8.3442e-1 (1.03e-1) = | 8.6800e-1 (9.24e-2) = | 5.8099e-1 (4.01e-2) + | 8.8572e-1 (2.55e-1) |
| | 200 | 1.1746e+1 (2.23e+0) − | 6.6275e-1 (1.67e-1) + | 1.3302e+0 (1.98e-1) = | 1.1931e+0 (4.66e-2) − | 1.2240e+0 (2.13e-1) = | 1.2372e+0 (1.78e-2) = | 1.0678e+0 (1.18e-1) = | 1.1797e+0 (2.41e-1) |
| | 500 | 1.5352e+2 (2.40e+1) − | 6.2049e-1 (1.28e-2) + | 1.8037e+0 (1.50e+0) − | 1.4856e+0 (4.10e-1) − | 2.1469e+0 (3.94e-1) − | 1.2946e+0 (4.72e-4) + | 1.5033e+0 (4.15e-1) = | 1.3065e+0 (9.54e-2) |
| | 1000 | 3.7621e+1 (3.47e+0) − | 6.8950e-1 (2.33e-2) + | 2.5430e+0 (3.99e+0) = | 3.0117e+0 (7.72e-1) − | 1.1716e+1 (7.53e+0) − | 1.4087e+0 (1.67e-1) = | 1.3081e+0 (1.76e-3) + | 1.4272e+0 (1.86e-1) |
| LSMOP7 | 100 | 1.3050e+0 (1.02e+0) − | 7.9034e-1 (4.27e-2) − | 1.0585e+0 (1.87e-2) − | 7.5353e-1 (2.39e-1) = | 7.5719e-1 (5.90e-2) − | 9.4593e-1 (6.24e-6) − | 6.0690e-1 (7.66e-2) = | 6.3012e-1 (2.35e-1) |
| | 200 | 2.1849e+0 (1.91e+0) − | 8.6713e-1 (3.19e-2) − | 9.5672e-1 (7.99e-3) − | 8.4554e-1 (1.56e-1) − | 7.4424e-1 (6.44e-2) − | 9.4601e-1 (6.26e-5) − | 1.3908e+0 (5.27e-1) − | 6.4726e-1 (2.05e-1) |
| | 500 | 5.0901e+0 (4.03e+0) − | 8.6115e-1 (8.76e-3) + | 8.7973e-1 (2.61e-3) − | 9.4594e-1 (4.88e-6) − | 9.0559e-1 (6.33e-2) = | 9.4935e-1 (1.64e-2) − | 1.0876e+0 (7.11e-1) − | 8.7621e-1 (2.76e-1) |
| | 1000 | 1.1252e+0 (6.74e-1) = | 8.5202e-1 (3.13e-3) = | 8.5759e-1 (3.82e-4) = | 9.8402e-1 (4.13e-2) − | 8.8545e-1 (7.52e-2) = | 9.6495e-1 (6.09e-3) = | 8.9345e-1 (1.58e-1) = | 9.3203e-1 (3.28e-1) |
| LSMOP8 | 100 | 1.0353e-1 (1.54e-3) − | 3.5738e-1 (6.93e-3) − | 3.6556e-1 (1.32e-3) − | 4.7550e-2 (3.57e-3) = | 9.5868e-2 (9.06e-3) − | 9.3224e-2 (1.33e-2) − | 2.2047e-1 (8.39e-2) − | 7.7075e-2 (9.02e-2) |
| | 200 | 1.2043e-1 (1.85e-3) − | 3.0148e-1 (5.72e-2) − | 2.4996e-1 (5.17e-3) − | 8.7214e-2 (3.94e-3) − | 1.1314e-1 (6.06e-3) − | 1.0828e-1 (7.36e-3) − | 3.3807e-1 (3.96e-2) − | 5.0416e-2 (1.92e-3) |
| | 500 | 4.4806e-1 (4.34e-3) − | 3.0061e-1 (2.84e-2) − | 2.1755e-1 (6.47e-3) − | 4.5804e-1 (1.33e-2) − | 4.8354e-1 (4.94e-3) − | 1.4578e-1 (1.62e-2) − | 2.7093e-1 (5.37e-2) − | 7.9473e-2 (1.29e-2) |
| | 1000 | 2.1816e-1 (2.77e-3) − | 3.1282e-1 (8.67e-3) − | 2.0692e-1 (6.33e-3) − | 4.9312e-1 (4.56e-3) − | 4.9678e-1 (4.56e-3) − | 1.7892e-1 (9.63e-2) = | 2.2912e-1 (5.20e-3) − | 1.3681e-1 (3.02e-2) |
| LSMOP9 | 100 | 3.0817e-1 (6.45e-2) + | 1.5377e+0 (6.90e-4) − | 1.3840e+0 (9.29e-2) − | 5.5515e-1 (2.82e-1) + | 1.2405e+0 (3.25e-1) = | 4.4171e-1 (3.08e-1) + | 8.3216e-1 (4.66e-3) + | 1.0528e+0 (2.16e-1) |
| | 200 | 4.6947e-1 (2.25e-2) + | 1.5379e+0 (0.00e+0) − | 1.1900e+0 (1.07e-1) − | 6.2089e-1 (2.94e-1) + | 1.4638e+0 (3.60e-1) − | 7.7569e-1 (3.21e-1) + | 1.1574e+0 (1.35e-2) − | 1.1454e+0 (7.16e-5) |
| | 500 | 3.5212e+0 (4.40e-1) − | 1.3414e+0 (2.05e-1) = | 1.1528e+0 (5.91e-2) = | 7.4730e-1 (1.55e-1) + | 1.8111e+0 (3.52e-1) − | 7.7666e-1 (2.72e-1) + | 1.3083e+0 (1.85e-2) − | 1.0534e+0 (2.15e-1) |
| | 1000 | 7.3524e-1 (3.85e-2) + | 1.1445e+0 (2.75e-4) − | 1.1226e+0 (1.05e-1) = | 7.7981e-1 (3.30e-2) + | 5.9822e+0 (1.91e+0) − | 8.1590e-1 (2.79e-1) + | 1.0834e+0 (2.29e-1) = | 1.0999e+0 (1.59e-1) |
| +/−/= | | 5/28/3 | 5/26/5 | 0/30/6 | 6/24/6 | 1/27/8 | 6/24/6 | 3/26/7 | −− |
| *Best*/*All* | | 5/36 | 6/36 | 0/36 | 4/36 | 0/36 | 2/36 | 2/36 | 17/36 |

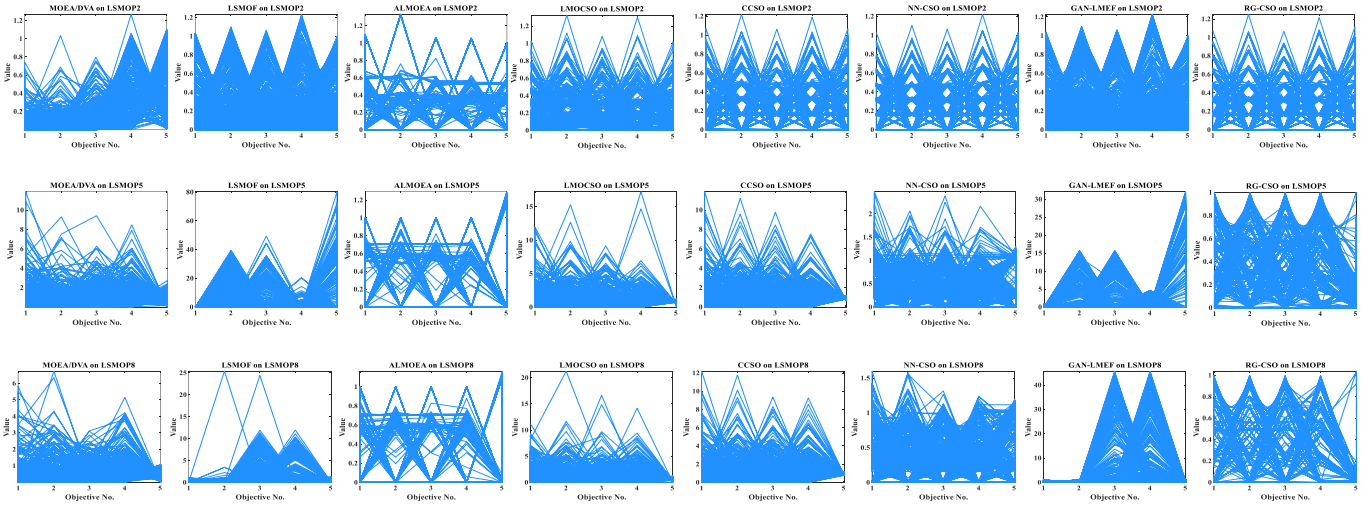
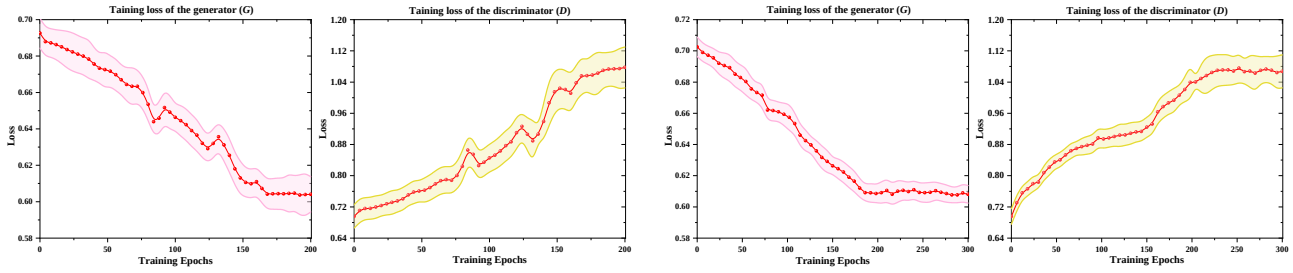



Fig. S2. Final solution sets obtained by RG-CSO and compared algorithms in solving LSMOP2, 5, 8 with $m = 5$, $n = 500$. The plots are based on the medium IGD over 30 independent runs. The objective values of the solutions are presented with the parallel coordinates.

TABLE S4
IGD RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON FIVE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 1.3672e-1 (6.64e-3) − | 4.2632e-1 (4.70e-2) − | 4.3587e-1 (1.67e-2) − | 1.2570e-1 (6.52e-3) − | 1.2779e-1 (8.86e-3) − | 3.2004e-1 (4.50e-2) − | 6.4617e-1 (5.76e-2) − | 1.1288e-1 (2.58e-2) |
| | 200 | 1.6889e-1 (4.82e-3) − | 7.2730e-1 (4.48e-2) − | 4.9005e-1 (1.28e-2) − | 2.0888e-1 (1.36e-2) − | 1.9431e-1 (9.77e-3) − | 4.4772e-1 (3.64e-2) − | 8.7248e-1 (1.54e-2) − | 1.6113e-1 (2.22e-1) |
| | 500 | 5.0917e-1 (2.25e-2) − | 8.9487e-1 (1.45e-2) − | 5.5654e-1 (8.76e-3) − | 4.3285e-1 (2.16e-2) − | 3.0941e-1 (1.03e-2) − | 6.0009e-1 (4.86e-2) − | 9.1328e-1 (7.49e-3) − | 2.1170e-1 (2.84e-2) |
| | 1000 | 2.2334e-1 (6.67e-3) + | 9.2144e-1 (1.57e-2) − | 5.8213e-1 (1.51e-2) − | 7.8435e-1 (3.90e-2) − | 4.5839e-1 (1.66e-2) − | 6.4924e-1 (4.05e-2) − | 1.0925e+0 (5.35e-1) − | 3.3267e-1 (4.52e-2) |
| LSMOP2 | 100 | 2.2529e-1 (9.97e-3) − | 4.1448e-1 (9.96e-3) − | 3.2630e-1 (6.81e-3) − | 1.3723e-1 (3.22e-3) = | 1.5295e-1 (5.10e-3) − | 1.8962e-1 (6.16e-3) − | 3.8404e-1 (5.37e-3) − | 1.3487e-1 (5.81e-3) |
| | 200 | 1.8996e-1 (6.92e-3) − | 2.5875e-1 (5.42e-3) − | 2.8763e-1 (5.65e-3) − | 1.5113e-1 (2.67e-3) = | 1.5952e-1 (2.13e-3) − | 1.6308e-1 (3.98e-3) − | 2.5177e-1 (2.28e-3) − | 1.4652e-1 (6.39e-3) |
| | 500 | 1.5862e-1 (5.91e-3) − | 1.5126e-1 (2.11e-3) − | 2.3088e-1 (3.23e-3) − | 1.2664e-1 (5.75e-4) − | 1.2769e-1 (9.77e-4) − | 1.2290e-1 (8.49e-4) − | 1.5636e-1 (2.45e-3) − | 1.2121e-1 (1.60e-3) |
| | 1000 | 1.4114e-1 (4.09e-3) − | 1.2578e-1 (1.84e-3) − | 2.1224e-1 (3.18e-3) − | 1.0965e-1 (3.18e-4) − | 1.1058e-1 (2.14e-4) − | 1.0696e-1 (4.82e-4) − | 1.2951e-1 (1.68e-3) − | 1.0355e-1 (9.64e-4) |
| LSMOP3 | 100 | 8.9723e-1 (1.06e-1) = | 9.5883e-1 (1.16e-16) − | 9.5882e-1 (9.40e-6) − | 5.3716e-1 (1.42e-1) − | 8.2576e-1 (1.99e-2) + | 8.9207e-1 (4.78e-2) − | 9.5882e-1 (2.53e-5) − | 5.1222e-1 (1.76e-1) |
| | 200 | 1.5925e+0 (4.46e-1) − | 9.5883e-1 (1.16e-16) − | 9.5882e-1 (9.80e-6) − | 7.9918e-1 (5.92e-2) = | 1.0518e+0 (4.78e-1) = | 9.3204e-1 (4.42e-2) = | 9.5883e-1 (1.16e-16) − | 8.3165e-1 (1.69e-1) |
| | 500 | 1.5809e+1 (5.77e+0) − | 9.5883e-1 (1.16e-16) − | 9.5882e-1 (7.35e-6) − | 4.0795e+0 (2.71e+0) − | 4.7914e+0 (1.32e+0) − | 9.5723e-1 (5.55e-3) − | 9.5883e-1 (1.16e-16) − | 9.1026e-1 (9.29e-2) |
| | 1000 | 3.7941e+0 (7.85e-1) − | 9.5883e-1 (1.16e-16) = | 9.5883e-1 (6.34e-6) = | 4.8812e+0 (3.07e+0) − | 7.2117e+0 (9.62e-1) − | 9.5883e-1 (1.16e-16) = | 2.6157e+0 (2.80e+0) − | 9.6958e-1 (2.45e-2) |
| LSMOP4 | 100 | 1.7196e-1 (7.26e-3) + | 5.2072e-1 (1.83e-2) − | 5.4181e-1 (8.76e-3) − | 2.2366e-1 (6.35e-3) − | 3.0150e-1 (2.42e-2) = | 3.2955e-1 (2.95e-2) − | 5.7137e-1 (2.25e-2) − | 2.1721e-1 (1.88e-2) |
| | 200 | 2.2028e-1 (1.10e-2) = | 4.3600e-1 (7.15e-3) − | 4.3241e-1 (1.72e-2) − | 2.8923e-1 (1.68e-2) − | 3.1195e-1 (8.56e-3) = | 3.0786e-1 (4.87e-3) = | 4.4197e-1 (1.10e-2) − | 2.2007e-1 (1.22e-2) |
| | 500 | 2.1961e-1 (8.38e-3) = | 3.0248e-1 (4.23e-3) − | 3.3725e-1 (5.60e-3) − | 2.1934e-1 (2.54e-3) − | 2.2007e-1 (2.48e-3) − | 2.0628e-1 (2.59e-3) = | 2.9114e-1 (4.02e-3) − | 2.1281e-1 (6.60e-3) |
| | 1000 | 1.6034e-1 (7.82e-3) = | 2.1730e-1 (1.59e-3) − | 2.8108e-1 (2.91e-3) − | 1.6808e-1 (1.51e-3) − | 1.6870e-1 (1.11e-3) − | 1.5764e-1 (1.61e-3) − | 2.1717e-1 (5.75e-3) − | 1.5313e-1 (1.88e-3) |
| LSMOP5 | 100 | 2.0492e-1 (4.39e-3) − | 5.1693e-1 (5.67e-2) − | 4.0559e-1 (9.50e-3) − | 2.5124e-1 (6.98e-2) = | 7.5392e-1 (4.86e-2) − | 8.3664e-1 (4.03e-1) − | 4.6514e-1 (4.14e-2) − | 1.9888e-1 (4.60e-2) |
| | 200 | 4.3334e-1 (1.44e-2) − | 4.6156e-1 (2.37e-2) − | 4.1064e-1 (5.73e-3) − | 3.9110e-1 (7.09e-2) − | 8.1441e-1 (3.18e-2) − | 2.8321e-1 (9.41e-3) − | 4.4611e-1 (1.63e-2) − | 2.5361e-1 (4.55e-2) |
| | 500 | 9.8263e-1 (4.45e-2) − | 4.7686e-1 (1.14e-2) − | 4.1626e-1 (5.21e-3) − | 7.7710e-1 (4.06e-2) − | 8.1371e-1 (2.09e-2) − | 3.0165e-1 (8.87e-3) = | 4.1655e-1 (2.67e-2) − | 3.0149e-1 (6.70e-2) |
| | 1000 | 5.5873e-1 (4.13e-3) − | 3.7928e-1 (2.82e-2) + | 4.1627e-1 (4.59e-3) = | 1.4901e+0 (7.65e-2) − | 9.5004e-1 (5.52e-2) − | 3.1423e-1 (6.53e-3) + | 7.6935e+0 (9.03e+0) = | 4.0658e-1 (3.50e-2) |
| LSMOP6 | 100 | 6.9105e-1 (8.68e-3) − | 7.0961e-1 (2.81e-2) − | 5.8988e-1 (1.42e-2) − | 5.2136e-1 (3.06e-2) − | 6.0214e-1 (2.72e-2) − | 5.5526e-1 (3.66e-2) − | 6.8936e-1 (9.67e-2) − | 3.8704e-1 (2.87e-2) |
| | 200 | 1.1150e+0 (6.19e-2) − | 1.4473e+0 (2.31e-1) − | 8.5314e-1 (1.70e-2) − | 8.6115e-1 (2.25e-1) = | 7.7125e-1 (7.15e-2) = | 7.8216e-1 (6.44e-2) = | 1.0899e+0 (1.27e-1) − | 7.3695e-1 (1.34e-1) |
| | 500 | 4.6493e+1 (9.92e+0) − | 1.7387e+0 (1.66e-1) − | 1.1698e+0 (2.45e-2) = | 1.2496e+0 (1.97e-1) − | 1.4148e+0 (1.58e-1) − | 1.1524e+0 (1.86e-1) − | 1.2594e+0 (7.35e-2) = | 1.1440e+0 (1.24e-1) |
| | 1000 | 1.6280e+1 (2.05e+0) − | 1.6903e+0 (2.88e-1) − | 1.2085e+0 (1.64e-2) = | 5.6315e+0 (3.59e+0) − | 3.8571e+0 (1.53e+0) − | 1.0728e+0 (1.26e-1) + | 6.2802e+0 (1.18e+1) − | 1.3940e+0 (2.42e-1) |
| LSMOP7 | 100 | 1.1501e+0 (1.72e-1) − | 1.0008e+0 (1.25e-1) − | 1.0619e+0 (3.19e-2) − | 7.2531e-1 (7.01e-2) = | 8.0275e-1 (2.42e-2) − | 1.0950e+0 (4.27e-2) − | 1.0153e+0 (1.51e-1) − | 6.9859e-1 (6.09e-2) |
| | 200 | 2.7804e+0 (1.27e+0) − | 9.9408e-1 (3.37e-2) = | 1.1400e+0 (4.70e-2) − | 9.5866e-1 (2.31e-1) = | 8.3068e-1 (3.31e-2) + | 1.1063e+0 (2.54e-3) − | 1.1401e+0 (1.33e-1) − | 9.4154e-1 (1.23e-1) |
| | 500 | 4.8922e+0 (3.85e+0) − | 1.2539e+0 (3.07e-2) = | 1.0574e+0 (1.63e-2) + | 1.5749e+0 (2.82e-1) − | 1.0793e+0 (1.85e-1) = | 1.0695e+0 (3.62e-2) + | 1.0853e+0 (8.50e-2) = | 1.2359e+0 (1.94e-1) |
| | 1000 | 1.3154e+0 (3.64e-1) = | 1.1772e+0 (1.52e-2) = | 1.0156e+0 (5.33e-3) + | 1.7085e+0 (8.75e-2) − | 1.4051e+0 (1.08e-1) − | 1.0452e+0 (4.84e-2) + | 1.1951e+0 (3.79e-1) = | 1.2663e+0 (3.23e-1) |
| LSMOP8 | 100 | 2.1530e-1 (5.75e-3) + | 3.8870e-1 (3.42e-2) − | 4.1176e-1 (4.19e-3) − | 2.5012e-1 (2.67e-2) = | 6.8408e-1 (1.05e-1) − | 2.6006e-1 (1.38e-2) = | 3.8097e-1 (2.75e-2) − | 2.6624e-1 (4.43e-2) |
| | 200 | 3.0337e-1 (1.00e-2) = | 3.7692e-1 (5.73e-2) − | 4.1635e-1 (4.05e-3) − | 3.5099e-1 (5.30e-2) − | 7.9086e-1 (2.12e-2) − | 3.4259e-1 (2.41e-1) = | 3.6028e-1 (2.20e-2) − | 2.8097e-1 (3.73e-2) |
| | 500 | 6.5304e-1 (5.00e-3) − | 5.3207e-1 (1.08e-1) − | 4.1173e-1 (4.06e-3) − | 7.9204e-1 (3.35e-2) − | 9.0025e-1 (3.51e-2) − | 3.4731e-1 (2.11e-1) = | 3.6331e-1 (3.37e-2) − | 3.1894e-1 (4.52e-2) |
| | 1000 | 3.8242e-1 (4.53e-3) − | 6.8223e-1 (4.00e-2) − | 3.8444e-1 (7.15e-3) − | 8.1791e-1 (1.99e-2) − | 8.4149e-1 (3.19e-2) − | 6.9602e-1 (3.72e-1) = | 5.5318e-1 (3.68e-1) − | 3.1312e-1 (5.15e-2) |
| LSMOP9 | 100 | 1.2566e+0 (1.81e-1) + | 2.9981e+0 (8.34e-3) − | 9.4898e-1 (3.23e-2) + | 6.1547e-1 (3.79e-2) + | 1.9387e+0 (7.30e-1) = | 6.9575e-1 (6.50e-2) + | 5.7913e-1 (8.05e-3) + | 1.7566e+0 (2.58e-1) |
| | 200 | 2.0328e+0 (2.77e-1) − | 3.0005e+0 (0.00e+0) − | 9.3546e-1 (2.60e-2) + | 6.6784e-1 (2.51e-1) + | 2.0817e+0 (5.59e-1) = | 7.5210e-1 (8.47e-2) + | 7.2563e-1 (4.73e-2) + | 1.8742e+0 (1.09e-2) |
| | 500 | 1.3083e+1 (1.09e+0) − | 3.0005e+0 (0.00e+0) − | 9.2244e-1 (2.00e-2) + | 6.1172e-1 (3.27e-2) + | 2.7733e+0 (9.30e-1) − | 7.7468e-1 (1.16e-1) + | 7.3446e-1 (4.28e-2) + | 1.7732e+0 (2.71e-1) |
| | 1000 | 2.9019e+0 (3.53e-1) − | 3.0005e+0 (0.00e+0) − | 8.7382e-1 (2.32e-2) − | 7.7788e-1 (2.69e-1) = | 2.9494e+1 (4.76e+0) − | 7.1435e-1 (5.21e-2) = | 9.8837e-1 (4.13e-1) = | 7.2314e-1 (1.08e-1) |
| +/−/= | | 4/26/6 | 1/31/4 | 5/27/4 | 3/24/9 | 2/27/7 | 7/18/11 | 3/28/5 | −− |
| *Best*/*All* | | 2/36 | 1/36 | 3/36 | 3/36 | 1/36 | 4/36 | 1/36 | 21/36 |




(a) $E_p = 200$ (b) $E_p = 300$

Fig. S3. Training loss trajectories of the generator and discriminator in GAN along with different training epochs in solving LSMOP6 with $m = 3$, $n = 500$.

TABLE S5
HV RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON TWO-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 5.7723e-1 (5.90e-4) − | 5.8076e-1 (1.97e-4) − | 2.4886e-1 (9.49e-3) − | 5.8455e-1 (1.17e-5) − | 5.8444e-1 (2.09e-5) − | 5.7260e-1 (2.22e-3) − | 5.8082e-1 (2.69e-4) = | 5.8820e-1 (3.86e-3) |
| | 200 | 5.5142e-1 (1.45e-3) − | 5.7678e-1 (9.21e-4) = | 2.1119e-1 (2.93e-3) − | 5.8418e-1 (4.41e-5) − | 5.8367e-1 (1.72e-4) − | 4.1525e-1 (1.95e-2) − | 5.7985e-1 (3.10e-4) − | 5.8590e-1 (1.37e-2) |
| | 500 | 3.0669e-1 (4.40e-3) − | 5.0074e-1 (3.46e-2) − | 1.8130e-1 (3.09e-3) − | 5.8077e-1 (1.05e-3) = | 4.2448e-1 (7.10e-2) − | 2.7111e-1 (1.08e-2) − | 5.6391e-1 (2.76e-3) − | 5.8080e-1 (4.79e-2) |
| | 1000 | 4.9289e-1 (8.37e-4) = | 2.5916e-1 (3.32e-2) − | 1.6836e-1 (3.12e-3) − | 3.1468e-1 (1.92e-2) − | 2.7515e-1 (3.11e-3) − | 2.4602e-1 (1.01e-2) − | 3.7834e-1 (5.90e-2) − | 4.9121e-1 (7.60e-2) |
| LSMOP2 | 100 | 3.6369e-1 (1.23e-3) − | 5.7605e-1 (7.35e-4) + | 4.7599e-1 (5.15e-3) − | 5.7739e-1 (3.04e-3) + | 5.7632e-1 (7.03e-3) + | 5.5641e-1 (5.19e-3) − | 5.7868e-1 (1.03e-3) + | 5.7049e-1 (1.28e-3) |
| | 200 | 4.1258e-1 (8.05e-4) − | 5.6800e-1 (1.09e-3) − | 5.1563e-1 (1.56e-3) − | 5.3893e-1 (1.78e-2) − | 5.4994e-1 (1.99e-2) − | 5.6150e-1 (1.13e-3) − | 5.7563e-1 (1.88e-3) + | 5.6838e-1 (1.23e-3) |
| | 500 | 5.0194e-1 (6.05e-4) − | 5.7062e-1 (3.75e-4) + | 5.5225e-1 (9.91e-4) − | 5.4501e-1 (8.37e-4) − | 5.4491e-1 (1.04e-3) − | 5.7140e-1 (4.17e-4) + | 5.7104e-1 (9.29e-4) + | 5.6883e-1 (1.63e-3) |
| | 1000 | 5.4159e-1 (3.83e-4) − | 5.7407e-1 (2.67e-4) = | 5.6581e-1 (6.60e-4) − | 5.5989e-1 (3.58e-4) − | 5.5996e-1 (2.69e-4) − | 5.7707e-1 (2.37e-4) + | 5.7383e-1 (4.80e-4) = | 5.7404e-1 (8.46e-4) |
| LSMOP3 | 100 | 1.2115e-5 (6.64e-5) − | 7.2336e-3 (2.00e-2) − | 0.0000e+0 (0.00e+0) − | 9.0911e-2 (6.14e-6) − | 9.0681e-2 (1.47e-4) − | 9.0909e-2 (7.38e-7) − | 5.7360e-2 (7.29e-2) − | 1.0507e-1 (3.67e-2) |
| | 200 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 9.0902e-2 (1.85e-6) − | 3.8572e-4 (2.11e-3) − | 1.2021e-2 (2.70e-2) − | 2.7127e-2 (3.99e-2) − | 9.1584e-2 (2.30e-2) |
| | 500 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 9.0864e-2 (8.42e-6) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 2.9521e-3 (1.62e-2) − | 9.0935e-2 (3.73e-2) |
| | 1000 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 9.0703e-2 (3.05e-5) + | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 9.0538e-2 (4.36e-2) |
| LSMOP4 | 100 | 5.1805e-1 (1.32e-2) − | 5.7594e-1 (2.04e-3) − | 3.5616e-1 (6.51e-3) − | 5.8112e-1 (1.52e-3) − | 5.7061e-1 (2.01e-3) − | 5.5924e-1 (5.30e-3) − | 5.7958e-1 (1.39e-3) − | 5.8890e-1 (4.18e-3) |
| | 200 | 5.1022e-1 (9.18e-3) − | 5.7242e-1 (6.08e-4) − | 4.0928e-1 (5.86e-3) − | 5.7334e-1 (1.34e-3) − | 5.7203e-1 (1.32e-3) − | 5.6286e-1 (4.27e-3) − | 5.7254e-1 (7.99e-4) − | 5.7918e-1 (5.26e-3) |
| | 500 | 5.2444e-1 (5.05e-4) − | 5.6573e-1 (2.32e-3) − | 4.6853e-1 (4.63e-3) − | 5.7026e-1 (1.73e-3) − | 5.6814e-1 (1.54e-3) − | 5.4371e-1 (2.72e-3) − | 5.7471e-1 (3.32e-4) = | 5.7416e-1 (1.19e-2) |
| | 1000 | 5.6240e-1 (5.13e-4) + | 5.5945e-1 (1.02e-3) + | 5.0889e-1 (3.24e-3) − | 5.3970e-1 (1.99e-3) − | 5.4375e-1 (1.51e-3) = | 5.5829e-1 (6.44e-4) + | 5.6356e-1 (1.21e-3) + | 5.4435e-1 (8.31e-3) |
| LSMOP5 | 100 | 3.3590e-1 (9.03e-4) − | 3.4585e-1 (3.06e-4) − | 9.0909e-2 (7.06e-17) − | 3.4870e-1 (6.05e-5) − | 3.4862e-1 (4.83e-5) − | 3.3153e-1 (4.63e-3) − | 3.4683e-1 (1.82e-4) − | 3.6128e-1 (5.77e-2) |
| | 200 | 2.7925e-1 (5.54e-3) − | 3.2217e-1 (8.67e-3) − | 9.0909e-2 (7.06e-17) − | 3.4781e-1 (2.30e-4) − | 3.3004e-1 (6.50e-2) − | 2.6566e-1 (2.12e-2) − | 3.2780e-1 (6.44e-2) − | 3.5442e-1 (5.60e-2) |
| | 500 | 0.0000e+0 (0.00e+0) − | 9.0909e-2 (7.06e-17) − | 9.0909e-2 (7.06e-17) − | 3.4188e-1 (7.75e-4) − | 1.5244e-1 (7.72e-2) − | 9.0905e-2 (2.45e-5) − | 2.6912e-1 (3.69e-2) − | 3.4921e-1 (3.42e-2) |
| | 1000 | 1.2453e-1 (1.91e-3) − | 9.0909e-2 (7.06e-17) − | 9.0909e-2 (7.06e-17) − | 1.9736e-1 (6.92e-2) − | 6.2054e-2 (1.81e-2) − | 9.0909e-2 (7.06e-17) − | 1.0863e-1 (1.10e-2) − | 2.8314e-1 (2.14e-2) |
| LSMOP6 | 100 | 0.0000e+0 (0.00e+0) − | 1.1030e-2 (1.50e-3) − | 0.0000e+0 (0.00e+0) − | 9.0112e-2 (2.01e-4) − | 8.8055e-2 (4.16e-4) − | 1.4163e-2 (3.96e-2) = | 6.8441e-2 (7.55e-2) − | 9.1771e-2 (5.42e-2) |
| | 200 | 0.0000e+0 (0.00e+0) = | 4.3765e-2 (6.23e-3) − | 6.5244e-3 (4.60e-3) + | 9.0064e-2 (9.17e-3) − | 6.2828e-2 (6.40e-3) − | 1.6281e-3 (8.91e-3) = | 1.1632e-2 (2.32e-2) − | 9.2007e-2 (1.09e-2) |
| | 500 | 0.0000e+0 (0.00e+0) = | 6.4774e-2 (5.11e-3) − | 5.1824e-2 (1.86e-3) + | 8.6562e-2 (3.50e-4) − | 3.7469e-2 (1.59e-2) − | 2.1947e-2 (1.65e-2) + | 1.5511e-2 (1.72e-2) − | 8.7461e-2 (3.46e-4) |
| | 1000 | 4.9515e-3 (8.30e-3) + | 9.0112e-2 (8.31e-3) + | 6.9029e-2 (1.23e-3) + | 6.7541e-2 (2.30e-3) + | 5.6125e-2 (9.77e-3) + | 3.8038e-2 (9.92e-3) + | 2.2435e-2 (2.08e-2) + | 0.0000e+0 (0.00e+0) |
| LSMOP7 | 100 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 1.6377e-3 (8.97e-3) − | 0.0000e+0 (0.00e+0) − | 3.0160e-3 (1.34e-2) = | 1.0924e-2 (2.69e-2) |
| | 200 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 1.1822e-2 (4.86e-2) − | 1.4031e-2 (3.72e-2) = | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 2.0090e-2 (8.84e-3) |
| | 500 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 1.5047e-2 (2.72e-2) = | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 1.5042e-2 (1.22e-4) |
| | 1000 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 5.7135e-3 (1.52e-2) |
| LSMOP8 | 100 | 2.8946e-1 (2.89e-3) − | 3.3274e-1 (4.25e-3) − | 9.0909e-2 (7.06e-17) − | 3.3888e-1 (2.72e-3) − | 3.3090e-1 (6.41e-3) − | 3.0302e-1 (8.56e-3) − | 3.3953e-1 (5.53e-3) = | 3.4409e-1 (5.47e-2) |
| | 200 | 2.7301e-1 (3.49e-3) − | 3.1090e-1 (5.73e-3) = | 9.0909e-2 (7.06e-17) − | 3.3057e-1 (5.28e-3) + | 3.0366e-1 (5.22e-3) = | 2.7832e-1 (4.29e-3) − | 3.2681e-1 (6.05e-3) + | 2.8348e-1 (5.86e-2) |
| | 500 | 1.8957e-2 (4.42e-3) − | 9.3982e-2 (1.68e-2) − | 8.9498e-2 (7.73e-3) − | 3.1610e-1 (8.29e-4) + | 3.1776e-1 (6.77e-4) + | 1.8321e-1 (8.82e-3) − | 2.8736e-1 (5.31e-2) + | 2.3093e-1 (5.85e-2) |
| | 1000 | 1.8871e-1 (1.36e-3) − | 9.0909e-2 (7.06e-17) − | 9.0909e-2 (7.06e-17) − | 3.0699e-1 (2.56e-3) + | 3.0909e-1 (1.53e-3) + | 9.0034e-2 (1.36e-3) − | 2.6507e-1 (3.46e-2) + | 2.1861e-1 (4.16e-2) |
| LSMOP9 | 100 | 1.8193e-1 (9.09e-3) − | 9.0909e-2 (7.06e-17) − | 4.8848e-2 (2.97e-2) − | 1.9234e-1 (5.57e-2) − | 7.4283e-2 (4.87e-3) − | 1.5702e-1 (6.42e-2) − | 9.0909e-2 (2.37e-16) − | 2.1129e-1 (5.87e-4) |
| | 200 | 1.7708e-1 (2.64e-3) − | 9.0909e-2 (7.06e-17) − | 4.0959e-2 (9.52e-3) − | 2.0004e-1 (3.10e-2) − | 7.1025e-2 (3.94e-3) − | 1.3759e-1 (3.88e-2) − | 1.6589e-1 (2.54e-2) − | 2.0190e-1 (1.27e-2) |
| | 500 | 3.6503e-2 (3.61e-3) − | 1.5154e-1 (1.16e-2) + | 5.5926e-2 (2.13e-2) − | 1.7165e-1 (3.19e-3) − | 6.0026e-2 (5.02e-3) − | 1.0506e-1 (2.61e-2) − | 1.6735e-1 (3.68e-3) − | 1.9295e-1 (1.19e-3) |
| | 1000 | 1.5314e-1 (3.04e-3) − | 1.4773e-1 (3.61e-3) + | 3.3643e-2 (1.07e-2) − | 1.9558e-1 (5.95e-4) + | 2.7805e-3 (9.43e-3) − | 9.7097e-2 (1.86e-2) − | 1.5755e-1 (1.35e-2) − | 1.8474e-1 (7.70e-4) |
| +/−/= | | 2/31/3 | 6/27/3 | 3/30/3 | 7/27/2 | 4/29/3 | 5/29/2 | 8/23/5 | −− |
| *Best*/*All* | | 1/36 | 1/36 | 0/36 | 4/36 | 2/36 | 2/36 | 4/36 | 22/36 |

TABLE S6
HV RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 8.3499e-1 (1.88e-3) − | 6.1059e-1 (2.17e-3) − | 3.5578e-1 (7.26e-3) − | 8.5359e-1 (5.01e-4) − | 8.4912e-1 (3.88e-3) + | 6.8296e-1 (7.31e-2) − | 7.3250e-1 (5.04e-2) − | 8.5433e-1 (6.50e-3) |
| | 200 | 7.6352e-1 (3.83e-3) − | 6.1308e-1 (2.77e-3) − | 3.2022e-1 (3.74e-3) − | 8.2729e-1 (1.14e-2) = | 7.5203e-1 (2.69e-2) − | 4.8493e-1 (1.41e-2) − | 5.6982e-1 (2.60e-2) − | 8.2798e-1 (1.92e-2) |
| | 500 | 3.2172e-1 (7.70e-3) − | 6.1065e-1 (1.87e-3) − | 2.9131e-1 (6.43e-3) − | 5.4906e-1 (1.37e-2) − | 5.6808e-1 (7.43e-3) − | 3.8481e-1 (1.99e-2) − | 2.8164e-1 (3.58e-2) − | 6.7539e-1 (2.51e-2) |
| | 1000 | 7.0519e-1 (3.92e-3) + | 5.6908e-1 (1.47e-2) + | 2.8292e-1 (3.70e-3) − | 1.8789e-1 (2.25e-2) − | 3.6218e-1 (1.15e-2) − | 3.6000e-1 (3.96e-2) − | 1.6542e-1 (7.05e-3) − | 5.1457e-1 (2.34e-2) |
| LSMOP2 | 100 | 7.1772e-1 (3.46e-3) − | 7.1653e-1 (6.86e-3) − | 7.1940e-1 (5.39e-3) − | 8.2930e-1 (1.22e-3) − | 8.1319e-1 (3.33e-3) − | 8.0467e-1 (3.42e-3) − | 6.6317e-1 (6.78e-3) − | 8.3445e-1 (3.18e-3) |
| | 200 | 7.5180e-1 (3.65e-3) − | 7.3259e-1 (2.57e-3) − | 7.6970e-1 (3.02e-3) − | 8.1720e-1 (9.08e-4) − | 8.1155e-1 (1.12e-3) − | 8.2536e-1 (1.86e-3) − | 7.5240e-1 (2.66e-3) − | 8.3072e-1 (3.24e-3) |
| | 500 | 8.0265e-1 (3.49e-3) − | 7.9922e-1 (2.62e-3) − | 8.0809e-1 (2.14e-3) − | 8.3457e-1 (2.25e-4) − | 8.3373e-1 (3.43e-4) − | 8.4121e-1 (8.58e-4) − | 8.1130e-1 (1.08e-3) − | 8.4290e-1 (3.44e-3) |
| | 1000 | 8.2512e-1 (2.13e-3) − | 8.2241e-1 (1.58e-3) − | 8.2182e-1 (1.38e-3) − | 8.4449e-1 (1.78e-4) − | 8.4403e-1 (1.92e-4) + | 8.4823e-1 (2.96e-4) + | 8.2890e-1 (1.14e-3) − | 8.4774e-1 (2.48e-3) |
| LSMOP3 | 100 | 6.9611e-2 (4.44e-2) − | 4.6277e-1 (1.50e-1) = | 9.0911e-2 (4.32e-6) − | 3.9789e-1 (6.15e-2) − | 9.4989e-2 (4.85e-3) − | 9.1931e-2 (1.97e-3) − | 1.7156e-1 (1.17e-1) = | 4.5745e-1 (1.18e-1) |
| | 200 | 0.0000e+0 (0.00e+0) − | 3.2002e-1 (1.28e-1) + | 9.0911e-2 (4.32e-6) − | 1.9969e-1 (5.21e-2) = | 3.7880e-2 (4.68e-2) − | 9.1005e-2 (2.89e-4) − | 9.0913e-2 (5.74e-6) − | 1.7558e-1 (7.47e-2) |
| | 500 | 0.0000e+0 (0.00e+0) − | 1.6758e-1 (4.86e-2) = | 9.0911e-2 (4.29e-6) − | 4.5751e-2 (4.78e-2) − | 0.0000e+0 (0.00e+0) − | 9.0912e-2 (1.14e-5) − | 9.0909e-2 (0.00e+0) − | 1.3390e-1 (4.52e-2) |
| | 1000 | 0.0000e+0 (0.00e+0) − | 9.0976e-2 (2.30e-5) − | 9.0912e-2 (4.86e-6) = | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) − | 9.0923e-2 (3.66e-5) = | 9.0909e-2 (0.00e+0) − | 1.1217e-1 (3.21e-2) |
| LSMOP4 | 100 | 7.2774e-1 (1.91e-2) − | 6.0501e-1 (3.23e-2) − | 4.2442e-1 (2.18e-2) − | 8.1710e-1 (1.43e-2) − | 7.7980e-1 (1.99e-2) = | 7.0520e-1 (1.67e-2) − | 6.2333e-1 (1.03e-2) − | 8.3778e-1 (3.11e-2) |
| | 200 | 7.2363e-1 (6.62e-3) − | 6.0820e-1 (7.37e-3) − | 5.9060e-1 (4.38e-3) − | 7.6034e-1 (1.40e-2) + | 7.5248e-1 (1.18e-2) = | 7.1783e-1 (7.39e-3) − | 5.6031e-1 (9.67e-3) − | 7.4502e-1 (1.15e-2) |
| | 500 | 7.6483e-1 (3.95e-3) − | 6.7311e-1 (4.57e-3) − | 7.0954e-1 (2.62e-3) − | 7.4317e-1 (3.50e-3) − | 7.5192e-1 (3.36e-3) − | 7.8885e-1 (2.90e-3) + | 6.7495e-1 (4.12e-3) − | 7.7939e-1 (6.29e-3) |
| | 1000 | 8.1788e-1 (2.27e-3) − | 7.3825e-1 (3.67e-3) − | 7.6801e-1 (2.83e-3) − | 7.8924e-1 (3.23e-4) − | 7.8955e-1 (4.41e-4) − | 8.2110e-1 (1.43e-3) − | 7.6005e-1 (1.39e-3) − | 8.2935e-1 (8.67e-3) |
| LSMOP5 | 100 | 5.2896e-1 (3.63e-3) − | 3.9603e-1 (1.36e-2) − | 3.4787e-1 (9.48e-5) − | 5.7711e-1 (3.82e-4) = | 3.9993e-1 (9.67e-2) − | 3.8861e-1 (3.28e-3) − | 4.8849e-1 (3.87e-2) − | 5.6535e-1 (4.52e-3) |
| | 200 | 2.4243e-1 (1.14e-2) − | 3.7647e-1 (2.13e-2) − | 3.4789e-1 (1.47e-4) − | 5.6515e-1 (3.55e-3) + | 1.6637e-1 (3.65e-2) − | 3.3914e-1 (3.85e-3) − | 4.0548e-1 (1.64e-3) − | 5.1147e-1 (5.40e-2) |
| | 500 | 0.0000e+0 (0.00e+0) − | 3.5487e-1 (1.28e-2) − | 3.4784e-1 (1.23e-4) − | 1.2464e-1 (1.00e-2) − | 8.9664e-2 (5.47e-3) − | 3.3711e-1 (2.37e-4) − | 3.4882e-1 (1.32e-4) − | 4.5450e-1 (3.13e-2) |
| | 1000 | 1.0712e-1 (2.34e-3) − | 3.4939e-1 (6.22e-5) − | 3.4774e-1 (1.30e-4) − | 2.4504e-4 (7.36e-4) − | 7.0975e-3 (4.40e-3) − | 3.3702e-1 (4.93e-4) − | 3.4880e-1 (1.00e-4) − | 3.7239e-1 (2.56e-2) |
| LSMOP6 | 100 | 0.0000e+0 (0.00e+0) − | 6.0246e-2 (4.20e-2) + | 0.0000e+0 (0.00e+0) − | 6.2288e-3 (8.73e-3) − | 4.6678e-3 (9.50e-3) = | 0.0000e+0 (0.00e+0) − | 2.0674e-1 (4.94e-2) + | 9.3590e-3 (2.12e-2) |
| | 200 | 0.0000e+0 (0.00e+0) = | 6.6684e-2 (3.15e-2) + | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 8.5105e-5 (2.95e-4) = | 0.0000e+0 (0.00e+0) = | 2.3511e-4 (8.14e-4) = | 0.0000e+0 (0.00e+0) |
| | 500 | 0.0000e+0 (0.00e+0) = | 5.0496e-2 (6.03e-3) + | 0.0000e+0 (0.00e+0) = | 6.6079e-3 (2.29e-2) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) |
| | 1000 | 0.0000e+0 (0.00e+0) = | 2.1103e-2 (7.11e-3) + | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) |
| LSMOP7 | 100 | 1.1269e-4 (3.90e-4) − | 2.9061e-2 (3.73e-2) − | 0.0000e+0 (0.00e+0) − | 1.0468e-1 (2.80e-2) = | 8.7757e-2 (3.28e-4) + | 9.0895e-2 (6.08e-6) − | 7.6352e-2 (6.37e-2) = | 1.2211e-1 (4.99e-2) |
| | 200 | 0.0000e+0 (0.00e+0) − | 1.5740e-3 (5.45e-3) − | 0.0000e+0 (0.00e+0) − | 8.6585e-2 (1.07e-2) − | 8.1331e-2 (1.93e-3) − | 9.0660e-2 (1.50e-4) = | 2.1797e-3 (5.32e-3) − | 8.8031e-2 (2.68e-2) |
| | 500 | 0.0000e+0 (0.00e+0) − | 1.2136e-4 (4.20e-4) − | 0.0000e+0 (0.00e+0) − | 9.0887e-2 (9.39e-6) = | 0.0000e+0 (0.00e+0) − | 4.8432e-2 (1.56e-2) − | 0.0000e+0 (0.00e+0) − | 8.5519e-2 (3.98e-2) |
| | 1000 | 1.6876e-2 (1.35e-2) = | 6.2135e-3 (8.81e-3) = | 0.0000e+0 (0.00e+0) − | 8.6360e-2 (1.56e-2) + | 0.0000e+0 (0.00e+0) − | 4.4220e-2 (3.83e-3) + | 2.6584e-2 (9.18e-3) = | 3.0521e-2 (3.26e-2) |
| LSMOP8 | 100 | 4.3717e-1 (6.24e-3) − | 3.9002e-1 (1.04e-2) − | 3.5738e-1 (5.87e-3) − | 5.4257e-1 (4.11e-3) − | 4.5829e-1 (1.61e-2) − | 4.7493e-1 (1.61e-2) − | 4.1663e-1 (3.03e-2) − | 5.4350e-1 (5.47e-2) |
| | 200 | 4.0601e-1 (5.99e-3) − | 3.6135e-1 (2.61e-2) − | 3.8991e-1 (2.37e-3) − | 4.7758e-1 (8.22e-3) − | 4.3093e-1 (8.92e-3) − | 4.5013e-1 (7.02e-3) − | 3.9269e-1 (7.61e-3) − | 5.4565e-1 (2.39e-3) |
| | 500 | 1.3705e-1 (2.31e-3) − | 3.1806e-1 (3.57e-2) − | 4.1825e-1 (3.36e-3) − | 1.4186e-1 (1.14e-2) − | 1.2236e-1 (4.37e-3) − | 4.2561e-1 (1.05e-2) − | 3.9707e-1 (3.58e-3) − | 4.8518e-1 (1.82e-2) |
| | 1000 | 3.0123e-1 (2.77e-3) − | 2.7304e-1 (8.97e-3) − | 4.2850e-1 (2.50e-3) = | 1.1495e-1 (3.58e-3) − | 1.1376e-1 (3.68e-3) − | 4.0293e-1 (8.79e-2) = | 4.0209e-1 (1.71e-3) − | 4.1426e-1 (4.20e-2) |
| LSMOP9 | 100 | 1.7171e-1 (1.53e-2) + | 9.0935e-2 (9.01e-5) − | 2.3292e-2 (4.75e-3) − | 1.5616e-1 (7.64e-2) = | 2.7647e-2 (1.66e-2) − | 1.5799e-1 (2.59e-2) = | 1.9428e-1 (8.53e-4) + | 1.5433e-1 (1.62e-2) |
| | 200 | 1.0754e-1 (6.78e-3) − | 9.0909e-2 (0.00e+0) − | 3.5279e-2 (1.17e-2) − | 1.1003e-1 (5.24e-2) − | 7.8261e-3 (4.51e-3) − | 1.7311e-1 (2.11e-2) + | 1.4404e-1 (3.36e-3) − | 1.4736e-1 (9.59e-6) |
| | 500 | 0.0000e+0 (0.00e+0) − | 1.1931e-1 (2.97e-2) = | 4.0769e-2 (1.48e-2) − | 7.4502e-2 (2.79e-2) − | 2.7171e-3 (2.98e-3) − | 1.5483e-1 (2.03e-2) = | 1.1212e-1 (3.16e-3) − | 1.5409e-1 (1.57e-2) |
| | 1000 | 4.5087e-2 (1.56e-3) − | 1.4770e-1 (4.82e-6) − | 4.0745e-2 (1.21e-2) − | 6.4894e-2 (1.41e-3) − | 0.0000e+0 (0.00e+0) − | 1.5799e-1 (4.49e-2) + | 1.3531e-1 (2.04e-2) − | 1.5053e-1 (1.14e-2) |
| +/−/= | | 2/30/4 | 6/26/4 | 0/31/5 | 3/24/9 | 3/27/6 | 5/23/8 | 2/28/6 | −− |
| *Best*/*All* | | 1/36 | 6/36 | 1/36 | 5/36 | 0/36 | 6/36 | 2/36 | 15/36 |

TABLE S7
HV RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON FIVE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 100 | 9.2805e-1 (5.48e-3) − | 6.2823e-1 (7.25e-2) − | 5.8604e-1 (3.75e-2) − | 9.6641e-1 (3.81e-3) − | 9.6580e-1 (5.14e-3) = | 7.5314e-1 (6.70e-2) − | 2.7132e-1 (5.54e-2) − | 9.6948e-1 (3.58e-2) |
| | 200 | 8.8844e-1 (8.56e-3) − | 2.4188e-1 (3.29e-2) − | 5.0000e-1 (2.42e-2) − | 8.4930e-1 (3.77e-2) − | 8.9211e-1 (1.22e-2) = | 5.7248e-1 (4.42e-2) − | 9.6691e-2 (7.20e-3) − | 8.8927e-1 (2.33e-1) |
| | 500 | 4.1386e-1 (2.68e-2) − | 9.1220e-2 (7.03e-4) − | 4.3119e-1 (1.71e-2) − | 5.4618e-1 (3.60e-2) − | 7.3077e-1 (1.16e-2) − | 3.5243e-1 (4.50e-2) − | 9.0842e-2 (2.31e-4) − | 8.6941e-1 (3.63e-2) |
| | 1000 | 8.2067e-1 (1.38e-2) + | 9.0909e-2 (0.00e+0) − | 3.9169e-1 (2.02e-2) − | 1.4472e-1 (3.12e-2) − | 5.1443e-1 (2.27e-2) − | 2.9180e-1 (4.11e-2) − | 8.3333e-2 (2.62e-2) − | 7.1475e-1 (5.44e-2) |
| LSMOP2 | 100 | 8.2262e-1 (1.18e-2) − | 6.7086e-1 (1.08e-2) − | 7.7626e-1 (1.95e-2) − | 9.5989e-1 (1.92e-3) = | 9.5594e-1 (2.20e-3) − | 9.2715e-1 (6.46e-3) − | 7.2021e-1 (5.50e-3) − | 9.5917e-1 (4.26e-3) |
| | 200 | 8.6365e-1 (6.18e-3) − | 8.5836e-1 (4.17e-3) − | 8.4999e-1 (9.85e-3) − | 9.5229e-1 (1.67e-3) − | 9.5127e-1 (1.05e-3) − | 9.4727e-1 (2.20e-3) − | 8.7940e-1 (2.53e-3) − | 9.5742e-1 (3.76e-3) |
| | 500 | 8.9635e-1 (6.41e-3) − | 9.5393e-1 (1.42e-3) − | 9.0724e-1 (5.97e-3) − | 9.6624e-1 (4.87e-4) − | 9.6605e-1 (5.12e-4) = | 9.6867e-1 (5.20e-4) + | 9.5190e-1 (1.14e-3) − | 9.6685e-1 (2.42e-3) |
| | 1000 | 9.1926e-1 (6.79e-3) − | 9.6755e-1 (5.22e-4) + | 9.2644e-1 (5.36e-3) − | 9.7429e-1 (1.57e-4) = | 9.7388e-1 (1.78e-4) = | 9.7555e-1 (2.64e-4) + | 9.6554e-1 (9.84e-4) = | 9.7450e-1 (3.03e-3) |
| LSMOP3 | 100 | 8.6568e-2 (5.48e-2) = | 9.0909e-2 (0.00e+0) − | 9.0912e-2 (4.34e-6) − | 4.6241e-1 (1.23e-1) + | 1.0650e-1 (1.44e-2) − | 9.5075e-2 (5.94e-3) = | 9.0909e-2 (0.00e+0) − | 1.6778e-1 (1.80e-1) |
| | 200 | 2.2708e-4 (7.87e-4) − | 9.0909e-2 (0.00e+0) − | 9.0913e-2 (3.87e-6) − | 1.3202e-1 (4.66e-2) = | 8.5463e-2 (2.70e-2) − | 9.4512e-2 (1.07e-2) − | 9.0909e-2 (0.00e+0) − | 2.0545e-1 (1.72e-1) |
| | 500 | 0.0000e+0 (0.00e+0) − | 9.0909e-2 (0.00e+0) − | 9.0912e-2 (4.68e-6) − | 3.1248e-2 (4.63e-2) − | 7.5758e-3 (2.62e-2) − | 9.0932e-2 (7.98e-5) − | 9.0909e-2 (0.00e+0) − | 1.2727e-1 (8.33e-2) |
| | 1000 | 0.0000e+0 (0.00e+0) − | 9.0909e-2 (0.00e+0) = | 9.0910e-2 (2.19e-6) = | 1.5152e-2 (3.54e-2) − | 0.0000e+0 (0.00e+0) − | 9.0909e-2 (0.00e+0) = | 6.0606e-2 (4.48e-2) = | 9.6639e-2 (2.81e-2) |
| LSMOP4 | 100 | 8.8541e-1 (1.04e-2) − | 4.7225e-1 (2.66e-2) − | 4.5039e-1 (2.45e-2) − | 8.9239e-1 (1.46e-2) − | 8.4855e-1 (2.50e-2) − | 7.8837e-1 (3.34e-2) − | 4.1143e-1 (3.48e-2) − | 8.9571e-1 (1.55e-2) |
| | 200 | 8.2938e-1 (1.57e-2) − | 6.0921e-1 (5.09e-3) − | 6.1988e-1 (3.04e-2) − | 8.2416e-1 (1.71e-2) − | 8.1557e-1 (1.43e-2) − | 8.0487e-1 (5.67e-3) − | 6.1830e-1 (1.33e-2) − | 8.5328e-1 (1.67e-2) |
| | 500 | 8.2849e-1 (1.04e-2) − | 8.0582e-1 (3.72e-3) − | 7.5831e-1 (1.22e-2) − | 8.9112e-1 (3.13e-3) − | 8.9621e-1 (2.83e-3) + | 9.1169e-1 (1.75e-3) + | 8.2538e-1 (4.27e-3) − | 8.9221e-1 (1.09e-2) |
| | 1000 | 8.9897e-1 (8.57e-3) − | 8.9431e-1 (2.83e-3) − | 8.4740e-1 (4.68e-3) − | 9.3863e-1 (1.36e-3) + | 9.3931e-1 (6.19e-4) + | 9.4888e-1 (1.32e-3) + | 9.0490e-1 (4.50e-3) − | 9.1485e-1 (7.19e-3) |
| LSMOP5 | 100 | 6.6776e-1 (8.18e-3) − | 4.7144e-1 (7.57e-2) − | 6.4923e-1 (1.32e-2) − | 6.6771e-1 (9.21e-2) − | 1.0786e-1 (3.15e-2) − | 2.8741e-1 (2.96e-1) − | 5.7442e-1 (3.25e-2) − | 7.6962e-1 (1.70e-2) |
| | 200 | 3.1809e-1 (9.77e-3) − | 5.4162e-1 (1.53e-2) − | 6.3770e-1 (1.61e-2) − | 4.7490e-1 (7.98e-2) − | 9.0931e-2 (8.13e-4) − | 7.2977e-1 (8.99e-4) = | 5.8380e-1 (1.44e-2) − | 7.3323e-1 (1.47e-2) |
| | 500 | 2.4051e-4 (6.04e-4) − | 5.1943e-1 (2.56e-2) − | 6.2508e-1 (1.05e-2) = | 9.5732e-2 (2.95e-2) − | 8.5491e-2 (1.47e-3) − | 7.1664e-1 (3.18e-3) = | 5.9197e-1 (2.61e-2) − | 7.2340e-1 (4.22e-2) |
| | 1000 | 1.7034e-1 (5.55e-3) − | 6.3641e-1 (1.22e-2) = | 6.2942e-1 (9.34e-3) = | 0.0000e+0 (0.00e+0) − | 1.3775e-2 (9.15e-3) − | 7.0785e-1 (2.43e-3) + | 3.5859e-1 (3.17e-1) = | 6.0791e-1 (6.12e-2) |
| LSMOP6 | 100 | 1.2984e-1 (2.24e-3) − | 1.8070e-1 (3.10e-2) − | 2.9423e-1 (1.73e-2) − | 3.1535e-1 (1.33e-2) − | 2.3338e-1 (2.72e-2) − | 2.5821e-1 (2.60e-2) − | 2.3724e-1 (6.77e-2) − | 3.9255e-1 (2.96e-2) |
| | 200 | 9.1440e-4 (3.07e-3) − | 0.0000e+0 (0.00e+0) − | 9.9396e-3 (6.43e-3) − | 9.5811e-2 (5.69e-2) = | 1.1315e-1 (3.47e-2) + | 4.9411e-2 (4.01e-2) = | 6.4378e-4 (8.16e-4) − | 6.2583e-2 (6.82e-2) |
| | 500 | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 4.8761e-5 (7.54e-5) = | 6.7942e-4 (2.35e-3) = | 5.1525e-4 (1.78e-3) = | 7.8302e-5 (6.88e-5) − | 0.0000e+0 (0.00e+0) = | 9.5414e-5 (6.90e-5) |
| | 1000 | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 3.5610e-5 (5.35e-5) = | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 1.2526e-3 (4.24e-3) + | 0.0000e+0 (0.00e+0) = | 4.2750e-7 (1.25e-6) |
| LSMOP7 | 100 | 0.0000e+0 (0.00e+0) − | 1.4743e-2 (2.76e-2) − | 4.3216e-4 (3.64e-4) − | 1.2555e-1 (3.03e-2) + | 8.9929e-2 (2.49e-4) + | 8.3690e-2 (2.50e-2) = | 2.3322e-3 (8.08e-3) − | 8.0462e-2 (2.73e-2) |
| | 200 | 0.0000e+0 (0.00e+0) − | 2.0606e-2 (2.23e-2) = | 5.6706e-5 (9.98e-5) − | 4.6928e-2 (4.49e-2) = | 8.5761e-2 (9.06e-4) − | 9.0892e-2 (1.90e-5) = | 0.0000e+0 (0.00e+0) − | 9.8736e-2 (4.06e-2) |
| | 500 | 0.0000e+0 (0.00e+0) = | 0.0000e+0 (0.00e+0) = | 1.9224e-6 (4.35e-6) = | 0.0000e+0 (0.00e+0) = | 2.1243e-2 (2.03e-2) = | 8.3520e-2 (3.33e-3) − | 4.3452e-4 (1.51e-3) = | 8.4888e-3 (1.93e-2) |
| | 1000 | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) = | 6.0257e-6 (1.16e-5) = | 0.0000e+0 (0.00e+0) − | 0.0000e+0 (0.00e+0) = | 5.6990e-2 (1.04e-2) + | 0.0000e+0 (0.00e+0) − | 2.2918e-2 (3.41e-2) |
| LSMOP8 | 100 | 6.3509e-1 (9.01e-3) − | 5.7217e-1 (4.65e-2) − | 6.4187e-1 (1.11e-2) − | 6.3949e-1 (4.96e-2) − | 1.3571e-1 (7.51e-2) − | 7.3381e-1 (4.01e-3) = | 5.3327e-1 (3.97e-2) − | 7.4083e-1 (1.71e-2) |
| | 200 | 4.7034e-1 (1.31e-2) − | 5.5892e-1 (8.80e-2) − | 6.2014e-1 (1.19e-2) − | 4.5572e-1 (4.86e-2) − | 8.7092e-2 (1.95e-3) − | 6.7517e-1 (1.84e-1) − | 5.4865e-1 (5.50e-2) − | 7.1643e-1 (1.68e-2) |
| | 500 | 1.2943e-1 (3.04e-3) − | 3.1231e-1 (1.56e-1) − | 6.3119e-1 (9.32e-3) − | 5.9508e-2 (1.40e-2) − | 4.9368e-2 (1.25e-2) − | 6.5699e-1 (1.84e-1) − | 5.4099e-1 (6.78e-2) − | 6.8046e-1 (2.86e-2) |
| | 1000 | 3.6142e-1 (4.39e-3) − | 1.9140e-1 (9.17e-3) − | 6.6503e-1 (8.83e-3) = | 3.6950e-2 (8.34e-3) − | 3.4868e-2 (5.41e-3) − | 3.5242e-1 (3.27e-1) = | 4.0228e-1 (2.47e-1) − | 6.4839e-1 (4.03e-2) |
| LSMOP9 | 100 | 4.8164e-2 (1.66e-2) − | 9.1114e-2 (7.11e-4) − | 7.4655e-3 (3.02e-3) − | 4.2749e-2 (1.53e-2) − | 3.8597e-3 (3.55e-3) − | 7.7243e-2 (5.38e-2) − | 2.1276e-1 (3.31e-3) + | 1.8530e-1 (4.88e-3) |
| | 200 | 2.6328e-3 (2.68e-3) − | 9.0909e-2 (0.00e+0) − | 8.2480e-3 (2.55e-3) − | 5.0544e-2 (6.21e-3) − | 1.8726e-3 (1.29e-3) − | 8.6445e-2 (6.50e-2) − | 1.4580e-1 (2.26e-2) − | 1.8339e-1 (6.96e-4) |
| | 500 | 0.0000e+0 (0.00e+0) − | 9.0909e-2 (0.00e+0) − | 8.5516e-3 (2.26e-3) − | 4.3743e-2 (3.95e-3) − | 3.2508e-4 (1.90e-4) − | 9.6710e-2 (6.02e-2) − | 1.4794e-1 (2.16e-2) − | 1.8575e-1 (8.73e-3) |
| | 1000 | 1.4245e-4 (1.03e-4) − | 9.0909e-2 (0.00e+0) − | 1.2874e-2 (3.48e-3) − | 4.1937e-2 (6.44e-3) − | 0.0000e+0 (0.00e+0) − | 1.3414e-1 (3.11e-2) − | 1.0281e-1 (1.07e-1) = | 1.8177e-1 (1.91e-2) |
| +/−/= | | 1/31/4 | 1/29/6 | 0/28/8 | 3/25/8 | 4/25/7 | 7/20/9 | 1/28/7 | −− |
| *Best*/*All* | | 1/36 | 0/36 | 1/36 | 4/36 | 2/36 | 8/36 | 1/36 | 19/36 |

TABLE S8
IGD RESULTS OBTAINED BY RG-CSO AND ITS SEVEN VARIANTS ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. THE SYMBOLS "+/−/=" INDICATE THAT THE VARIANT IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | RG-CSO$_A$ | RG-CSO$_B$ | RG-CSO$_C$ | RG-CSO$_D$ | RG-CSO$_E$ | RG-CSO$_F$ | RG-CSO$_G$ | RG-CSO |
|---|---|---|---|---|---|---|---|---|
| LSMOP1 | 3.0089e-1 (3.08e-1) − | 2.3196e-1 (2.21e-2) − | 3.5327e-1 (2.90e-2) − | 1.8271e-1 (1.13e-1) − | 2.3144e-1 (1.83e-2) − | 1.5430e-1 (9.18e-2) − | 2.0104e-1 (5.93e-2) − | 1.5276e-1 (1.99e-2) |
| LSMOP2 | 4.3651e-2 (9.11e-4) − | 5.0694e-2 (2.89e-3) − | 4.4057e-2 (1.35e-3) − | 5.0203e-2 (2.71e-3) − | 3.7938e-2 (1.11e-3) − | 3.9229e-2 (5.87e-4) − | 3.6924e-2 (2.71e-3) = | 3.5923e-2 (2.58e-3) |
| LSMOP3 | 9.0738e-1 (3.66e-1) = | 8.5887e-1 (3.85e-3) − | 7.3275e-1 (2.43e-1) = | 8.6048e-1 (5.57e-4) − | 8.6068e-1 (9.75e-6) − | 8.6071e-1 (1.55e-5) − | 6.6332e-1 (1.36e-1) + | 7.3012e-1 (1.18e-1) |
| LSMOP4 | 1.3211e-1 (1.96e-3) − | 1.0013e-1 (2.59e-3) − | 1.3450e-1 (2.21e-3) − | 9.8496e-2 (2.11e-3) − | 8.9099e-2 (2.26e-3) = | 9.1684e-2 (1.84e-3) = | 9.1855e-2 (2.40e-3) − | 9.1169e-2 (4.41e-3) |
| LSMOP5 | 5.2387e-1 (1.10e-1) − | 2.3030e-1 (1.61e-1) = | 5.6746e-1 (2.96e-2) − | 2.8874e-1 (1.20e-1) = | 1.8172e-1 (1.31e-1) − | 1.5639e-1 (1.85e-2) − | 1.5588e-1 (7.70e-2) = | 1.5572e-1 (9.60e-2) |
| LSMOP6 | 1.6439e+0 (3.28e-1) − | 1.5491e+0 (3.53e-1) − | 1.6478e+0 (2.64e-1) = | 1.4387e+0 (6.49e-2) − | 1.2702e+0 (2.95e-3) − | 1.2695e+0 (1.64e-3) + | 1.3111e+0 (1.22e-1) − | 1.3065e+0 (9.54e-2) |
| LSMOP7 | 9.0689e-1 (1.23e-1) − | 1.4682e+0 (1.85e-2) − | 9.4594e-1 (7.86e-6) − | 1.2680e+0 (3.01e-1) − | 1.3083e+0 (1.61e-3) − | 1.3089e+0 (1.53e-3) − | 8.8744e-1 (6.53e-2) − | 8.7621e-1 (2.76e-1) |
| LSMOP8 | 4.3866e-1 (7.88e-3) − | 1.7160e-1 (1.44e-1) − | 4.7421e-1 (8.83e-3) − | 2.6482e-1 (1.03e-1) − | 9.9659e-2 (3.90e-2) − | 1.2954e-1 (6.18e-2) − | 1.2273e-1 (3.23e-2) − | 7.9473e-2 (1.29e-2) |
| LSMOP9 | 9.6384e-1 (1.52e-1) + | 1.1454e+0 (7.05e-5) = | 9.9619e-1 (1.59e-1) = | 9.2486e-1 (2.85e-1) + | 1.1454e+0 (3.25e-5) − | 1.1454e+0 (1.71e-5) = | 1.0902e+0 (1.75e-1) − | 1.0534e+0 (2.15e-1) |
| +/−/= | 1/7/1 | 0/7/2 | 0/6/3 | 1/7/1 | 0/8/1 | 1/6/2 | 1/6/2 | −− |
| *Best*/*All* | 0/9 | 0/9 | 0/9 | 1/9 | 1/9 | 1/9 | 1/9 | 5/9 |

TABLE S9
IGD AND RUNTIME RESULTS OBTAINED BY RG-CSO WITH DIFFERENT $t_r$ VALUES ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. $t_r$=100 IS USED AS THE COMPARED PARAMETER, AND THE SYMBOLS "+/−/=" INDICATE THAT RG-CSO WITH OTHER $t_r$ VALUES IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO WITH $t_r$=100, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | $t_r = 50$ | | $t_r = 150$ | | $t_r = 200$ | | $t_r = 250$ | | $t_r = 100$ † | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) |
| LSMOP1 | 1.5131e-1 = | 292.90 | 1.8060e-1 − | 100.54 | 1.9158e-1 − | 95.84 | 2.3038e-1 − | 90.75 | 1.5276e-1 | 136.87 |
| LSMOP2 | 3.4345e-2 = | 195.21 | 3.7702e-2 = | 103.05 | 3.7874e-2 = | 95.83 | 3.7848e-2 = | 89.04 | 3.5923e-2 | 121.36 |
| LSMOP3 | 8.6072e-1 − | 4584.55 | 7.2947e-1 = | 1605.40 | 8.8735e-1 − | 1548.59 | 9.4953e-1 − | 1457.32 | 7.3012e-1 | 2305.40 |
| LSMOP4 | 8.7478e-2 + | 172.89 | 9.2010e-2 = | 103.63 | 9.2517e-2 = | 96.98 | 9.5574e-2 − | 92.84 | 9.1169e-2 | 120.41 |
| LSMOP5 | 8.1493e-2 + | 499.41 | 1.7600e-1 = | 295.86 | 2.2637e-1 − | 229.48 | 3.1442e-1 − | 184.56 | 1.5572e-1 | 400.88 |
| LSMOP6 | 1.2787e+0 + | 1135.26 | 1.4679e+0 − | 496.62 | 1.5729e+0 − | 454.91 | 1.7756e+0 − | 377.10 | 1.3065e+0 | 692.99 |
| LSMOP7 | 8.5396e-1 = | 1313.91 | 8.7630e-1 = | 438.03 | 8.8569e-1 = | 431.96 | 8.9263e-1 − | 404.74 | 8.7621e-1 | 647.07 |
| LSMOP8 | 1.0340e-1 = | 226.04 | 8.2996e-2 = | 121.44 | 8.7258e-2 − | 97.99 | 1.0493e-1 − | 93.91 | 7.9473e-2 | 149.17 |
| LSMOP9 | 1.0453e+0 = | 1096.50 | 1.1458e+0 = | 565.45 | 1.1555e+0 = | 479.44 | 1.4453e+0 − | 344.85 | 1.0534e+0 | 625.56 |
| +/−/= & Runtime Ratio | 3/1/5 | 1.83x | 0/2/7 | 0.74x | 0/5/4 | 0.68x | 0/8/1 | 0.60x | −− | −− |

† $t_r = 100$ is adopted in RG-CSO and is used as the compared parameter in the experiments.

TABLE S10
IGD AND RUNTIME RESULTS OBTAINED BY RG-CSO WITH DIFFERENT GAN TRAINING EPOCHS ($E_p$) ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. $E_p$=200 IS USED AS THE COMPARED PARAMETER, AND THE SYMBOLS "+/−/=" INDICATE THAT RG-CSO WITH OTHER $E_p$ VALUES IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO WITH $E_p$=200, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | $E_p = 50$ | | $E_p = 100$ | | $E_p = 300$ | | $E_p = 500$ | | $E_p = 200$ † | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) |
| LSMOP1 | 1.6508e-1 − | 73.78 | 1.5830e-1 = | 111.60 | 1.5463e-1 = | 208.33 | 1.4944e-1 = | 377.52 | 1.5276e-01 | 136.87 |
| LSMOP2 | 3.6692e-2 − | 68.59 | 3.8105e-2 − | 90.97 | 3.5743e-2 = | 157.26 | 3.5486e-2 = | 249.34 | 3.5923e-02 | 121.36 |
| LSMOP3 | 7.8966e-1 − | 795.03 | 7.3318e-1 − | 1463.25 | 7.3191e-1 = | 2549.19 | 7.3292e-1 = | 3414.51 | 7.3012e-01 | 2305.40 |
| LSMOP4 | 9.2538e-2 = | 68.34 | 9.1991e-2 = | 85.24 | 9.0522e-2 = | 157.21 | 9.0582e-2 = | 244.97 | 9.1169e-02 | 120.41 |
| LSMOP5 | 1.6994e-1 − | 171.76 | 1.6196e-1 − | 229.96 | 1.4098e-1 = | 502.45 | 1.2516e-1 = | 772.52 | 1.5572e-01 | 400.88 |
| LSMOP6 | 1.3974e+0 − | 261.10 | 1.3581e+0 − | 351.86 | 1.3193e+0 = | 1006.95 | 1.329e+0 = | 1190.92 | 1.3065e+0 | 692.99 |
| LSMOP7 | 9.9931e-1 − | 337.81 | 8.7747e-1 = | 439.03 | 7.2173e-1 + | 953.06 | 6.8324e-1 + | 1282.04 | 8.7621e-01 | 647.07 |
| LSMOP8 | 9.2203e-2 − | 78.63 | 8.4428e-2 = | 98.92 | 7.9818e-2 = | 214.66 | 7.9902e-2 = | 257.21 | 7.9473e-02 | 149.17 |
| LSMOP9 | 1.1456e+0 − | 244.67 | 1.0902e+0 = | 337.31 | 1.0008e+0 = | 904.84 | 6.5458e-1 + | 1354.86 | 1.0534e+0 | 625.56 |
| +/−/= & Runtime Ratio | 0/8/1 | 0.40x | 0/4/5 | 0.62x | 1/0/8 | 1.28x | 2/0/7 | 1.76x | −− | −− |

† $E_p = 200$ is adopted in RG-CSO and is used as the compared parameter in the experiments.

TABLE S11
IGD AND RUNTIME RESULTS OBTAINED BY RG-CSO WITH DIFFERENT NETWORK LAYERS ($d$) IN THE GAN MODEL ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. $d$=3 IS USED AS THE COMPARED PARAMETER, AND THE SYMBOLS "$+/-/=$" INDICATE THAT RG-CSO WITH OTHER $d$ VALUES IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO WITH $d$=3, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | $d=4$ | | $d=5$ | | $d=6$ | | $d=3$ † | |
|---|---|---|---|---|---|---|---|---|
| | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) | Mean IGD | Runtime (s) |
| LSMOP1 | 1.6683e-1 = | 139.00 | 1.6224e-1 = | 142.98 | 1.6087e-1 = | 150.35 | 1.5276e-01 | 136.87 |
| LSMOP2 | 3.8325e-2 − | 129.14 | 3.8091e-2 − | 133.91 | 3.9526e-2 − | 137.70 | 3.5923e-02 | 121.36 |
| LSMOP3 | 7.6653e-1 = | 2394.01 | 7.1687e-1 = | 2433.99 | 6.9007e-1 = | 2509.00 | 7.3012e-01 | 2305.40 |
| LSMOP4 | 9.0133e-2 = | 125.06 | 9.1649e-2 = | 129.05 | 9.5437e-2 = | 131.77 | 9.1169e-02 | 120.41 |
| LSMOP5 | 1.2998e-1 = | 412.09 | 1.6428e-1 = | 415.60 | 1.4276e-1 = | 419.91 | 1.5572e-01 | 400.88 |
| LSMOP6 | 1.3249e+0 = | 706.04 | 1.3030e+0 = | 722.91 | 1.3304e+0 = | 735.05 | 1.3065e+0 | 692.99 |
| LSMOP7 | 7.9280e-1 = | 659.00 | 7.5933e-1 = | 675.58 | 1.0478e+0 = | 687.30 | 8.7621e-01 | 647.07 |
| LSMOP8 | 8.1135e-2 = | 154.57 | 9.6910e-2 = | 158.85 | 9.2427e-2 = | 170.71 | 7.9473e-02 | 149.17 |
| LSMOP9 | 1.1455e+0 = | 632.38 | 1.1455e+0 = | 646.40 | 9.5802e-1 + | 658.04 | 1.0534e+0 | 625.56 |
| $+/-/=$ & Runtime Ratio | 0/1/8 | 1.03x | 0/1/8 | 1.05x | 1/1/7 | 1.08x | −− | −− |

† $d=3$ is adopted in RG-CSO and is used as the compared parameter in the experiments.

TABLE S12
IGD RESULTS OBTAINED BY RG-CSO WITH DIFFERENT LEARNING RATES ($\eta$) IN THE GAN MODEL ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. $\eta$=0.001 IS USED AS THE COMPARED PARAMETER, AND THE SYMBOLS "$+/-/=$" INDICATE THAT RG-CSO WITH OTHER $\eta$ VALUES IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO WITH $\eta$=0.001, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | $\eta=0.0001$ | $\eta=0.0005$ | $\eta=0.002$ | $\eta=0.005$ | $\eta=0.001$ † |
|---|---|---|---|---|---|
| LSMOP1 | 1.5516e-1 (1.78e-2) = | 1.5678e-1 (2.48e-2) = | 1.5956e-1 (2.01e-2) = | 1.6918e-1 (2.74e-2) − | 1.5276e-1 (1.99e-2) |
| LSMOP2 | 3.7247e-2 (1.74e-3) − | 3.7761e-2 (1.77e-3) − | 3.5567e-2 (2.70e-3) = | 3.8054e-2 (2.34e-3) = | 3.5923e-2 (2.58e-3) |
| LSMOP3 | 6.8224e-1 (1.10e-1) = | 7.9610e-1 (1.12e-1) = | 7.3765e-1 (1.59e-1) = | 6.6423e-1 (1.11e-1) = | 7.3012e-1 (1.18e-1) |
| LSMOP4 | 8.8363e-2 (3.18e-3) = | 9.1098e-2 (5.95e-3) = | 9.4142e-2 (7.86e-3) = | 9.9837e-2 (1.04e-2) − | 9.1169e-2 (4.41e-3) |
| LSMOP5 | 1.7494e-1 (1.32e-1) − | 1.6506e-1 (6.74e-2) − | 1.1866e-1 (2.37e-2) = | 1.5887e-1 (7.01e-2) − | 1.5572e-1 (9.60e-2) |
| LSMOP6 | 1.3613e+0 (1.88e-1) = | 1.3085e+0 (1.05e-1) = | 1.2886e+0 (1.35e-1) = | 1.2665e+0 (2.25e-1) = | 1.3065e+0 (9.54e-2) |
| LSMOP7 | 8.2291e-1 (5.12e-1) = | 8.7773e-1 (2.82e-1) = | 7.1614e-1 (1.08e-1) = | 8.5200e-1 (3.22e-1) = | 8.7621e-1 (2.76e-1) |
| LSMOP8 | 9.6642e-2 (2.44e-2) − | 9.7176e-2 (3.34e-2) − | 8.7569e-2 (1.56e-2) = | 8.9866e-2 (2.70e-2) − | 7.9473e-2 (1.29e-2) |
| LSMOP9 | 9.4869e-1 (2.88e-1) + | 1.1456e+0 (1.93e-4) = | 9.7945e-1 (2.67e-1) = | 1.0347e+0 (2.34e-1) = | 1.0534e+0 (2.15e-1) |
| $+/-/=$ | 1/3/5 | 0/3/6 | 0/0/9 | 0/4/5 | −− |

† $\eta=0.001$ is adopted in RG-CSO and is used as the compared parameter in the experiments.

TABLE S13
IGD RESULTS OBTAINED BY RG-CSO WITH DIFFERENT ACTIVATION FUNCTIONS IN THE OUTPUT LAYERS OF THE GENERATOR AND DISCRIMINATOR NETWORKS ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. ADOPTING THE SIGMOID FUNCTION IN BOTH GENERATOR AND DISCRIMINATOR IS USED AS THE COMPARED CONFIGURATION, AND THE SYMBOLS "$+/-/=$" INDICATE THAT RG-CSO WITH OTHER CONFIGURATIONS IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO THE COMPARED CONFIGURATION, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | ($G$: Sigmoid, $D$: LeakyRelu) | ($G$: Tanh, $D$: Sigmoid) | ($G$: Tanh, $D$: LeakyRelu) | ($G$: Sigmoid, $D$: Sigmoid) † |
|---|---|---|---|---|
| LSMOP1 | 1.5271e-1 (2.76e-2) = | 1.6253e-1 (2.63e-2) = | 1.6427e-1 (1.88e-2) = | 1.5276e-1 (1.99e-2) |
| LSMOP2 | 3.8165e-2 (2.37e-3) = | 3.7912e-2 (2.40e-3) = | 3.7712e-2 (1.86e-3) = | 3.5923e-2 (2.58e-3) |
| LSMOP3 | 7.8510e-1 (1.22e-1) = | 8.3179e-1 (9.11e-2) = | 8.0298e-1 (1.22e-1) = | 7.3012e-1 (1.18e-1) |
| LSMOP4 | 9.2910e-2 (5.35e-3) = | 9.3762e-2 (6.02e-3) = | 9.1945e-2 (5.12e-3) = | 9.1169e-2 (4.41e-3) |
| LSMOP5 | 1.6466e-1 (7.19e-2) = | 1.4444e-1 (7.57e-2) = | 2.1845e-1 (1.20e-1) = | 1.5572e-1 (9.60e-2) |
| LSMOP6 | 1.3433e+0 (1.38e-1) = | 1.2733e+0 (2.01e-3) = | 1.2651e+0 (2.93e-2) = | 1.3065e+0 (9.54e-2) |
| LSMOP7 | 8.0262e-1 (2.01e-1) = | 1.0839e+0 (8.90e-1) = | 7.3837e-1 (2.24e-1) = | 8.7621e-1 (2.76e-1) |
| LSMOP8 | 7.8415e-2 (1.14e-2) = | 7.9901e-2 (2.09e-2) = | 8.1309e-2 (1.82e-2) = | 7.9473e-2 (1.29e-2) |
| LSMOP9 | 1.0901e+0 (1.75e-1) = | 1.0341e+0 (2.33e-1) = | 1.0901e+0 (1.75e-1) = | 1.0534e+0 (2.15e-1) |
| $+/-/=$ | 0/0/9 | 0/0/9 | 0/0/9 | $--$ |

† The sigmoid function is adopted in both generator ($G$) and discriminator ($D$) networks and is used as the compared setting in the experiments.

TABLE S14
IGD RESULTS OBTAINED BY RG-CSO WITH DIFFERENT OPTIMIZERS IN THE GAN MODEL ON THREE-OBJECTIVE LSMOP1-LSMOP9 PROBLEMS WITH 500 DECISION VARIABLES OVER 30 INDEPENDENT RUNS. ADOPTING THE ADAM OPTIMIZER IS USED AS THE COMPARED CONFIGURATION, AND THE SYMBOLS "$+/-/=$" INDICATE THAT RG-CSO WITH OTHER OPTIMIZERS IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO WITH THE ADAM OPTIMIZER, RESPECTIVELY. THE BEST IGD RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY.

| Problems | SGD | AdamW | Adam † |
|---|---|---|---|
| LSMOP1 | 1.5721e-1 (2.76e-2) = | 1.6427e-1 (1.88e-2) = | 1.5276e-1 (1.99e-2) |
| LSMOP2 | 3.7998e-2 (1.68e-3) − | 3.7278e-2 (2.04e-3) = | 3.5923e-2 (2.58e-3) |
| LSMOP3 | 7.4530e-1 (1.23e-1) = | 7.2985e-1 (1.00e-1) = | 7.3012e-1 (1.18e-1) |
| LSMOP4 | 9.3522e-2 (6.09e-3) = | 9.2143e-2 (4.10e-3) = | 9.1169e-2 (4.41e-3) |
| LSMOP5 | 1.2757e-1 (2.22e-2) = | 1.7199e-1 (9.94e-2) = | 1.5572e-1 (9.60e-2) |
| LSMOP6 | 1.2691e+0 (1.70e-1) = | 1.3058e+0 (9.12e-2) = | 1.3065e+0 (9.54e-2) |
| LSMOP7 | 7.5156e-1 (7.94e-2) = | 7.8724e-1 (2.05e-1) = | 8.7621e-1 (2.76e-1) |
| LSMOP8 | 9.0400e-2 (1.48e-2) = | 8.7062e-2 (1.63e-2) = | 7.9473e-2 (1.29e-2) |
| LSMOP9 | 1.0901e+0 (1.75e-1) = | 1.0455e+0 (1.70e-4) = | 1.0534e+0 (2.15e-1) |
| $+/-/=$ | 0/1/8 | 0/0/9 | $--$ |

† The Adam optimizer is adopted in RG-CSO and is used as the compared setting in the experiments.

TABLE S15
HV RESULTS OBTAINED BY RG-CSO AND SEVEN COMPARATIVE LSMOEAS ON TEN REAL-WORLD CASES OVER 30 INDEPENDENT RUNS. THE SYMBOLS "$+/-/=$" INDICATE THAT THE COMPARED ALGORITHM IS SIGNIFICANTLY BETTER THAN, WORSE THAN, AND EQUAL TO RG-CSO, RESPECTIVELY. THE BEST RESULT OF EACH TEST INSTANCE IS HIGHLIGHTED IN GRAY, AND "*Best*" DENOTES THE TOTAL NUMBER OF THE BEST RESULTS OBTAINED BY EACH ALGORITHM.

| Problems | $m$ | $n$ | MOEA/DVA | LSMOF | ALMOEA | LMOCSO | CCSO | NN-CSO | GAN-LMEF | RG-CSO |
|---|---|---|---|---|---|---|---|---|---|---|
| TREE1 | 2 | 1500 | NaN (NaN) † | 8.3120e-1 (2.13e-4) − | NaN (NaN) | 8.4935e-1 (2.88e-5) = | 7.1738e-1 (1.75e-2) − | 8.4707e-1 (1.23e-4) + | 8.4205e-1 (1.31e-4) − | 8.4978e-1 (3.43e-4) |
| TREE2 | 2 | 1500 | NaN (NaN) | 8.5089e-1 (4.77e-5) − | NaN (NaN) | 8.5439e-1 (1.09e-4) − | 7.9846e-1 (1.24e-2) − | 8.4809e-1 (3.31e-4) − | 8.5416e-1 (3.17e-4) − | 8.5586e-1 (1.59e-4) |
| TREE3 | 2 | 1500 | NaN (NaN) | 8.8739e-1 (2.74e-6) = | NaN (NaN) | 8.4712e-1 (1.70e-3) − | NaN (NaN) | 8.0280e-1 (9.56e-3) − | 8.8725e-1 (7.31e-5) = | 8.8736e-1 (1.15e-2) |
| TREE4 | 2 | 1500 | NaN (NaN) | 9.5694e-1 (1.11e-5) + | NaN (NaN) | 9.1970e-1 (2.09e-3) − | 7.8908e-1 (4.59e-2) − | 8.2326e-1 (4.53e-3) − | 9.5692e-1 (2.45e-4) + | 9.5378e-1 (3.35e-3) |
| TREE5 | 2 | 1500 | NaN (NaN) | 9.2841e-1 (1.10e-4) − | NaN (NaN) | 9.3572e-1 (7.45e-5) − | NaN (NaN) | 9.2464e-1 (5.81e-4) − | 9.3362e-1 (4.44e-4) − | 9.3598e-1 (1.08e-3) |
| NN1 | 2 | 321 | 7.6925e-2 (5.19e-3) − | 3.3680e-1 (1.90e-2) − | 3.1721e-1 (1.39e-2) − | 3.5315e-1 (1.94e-2) = | 3.2629e-1 (6.13e-3) − | 3.2857e-1 (1.20e-2) − | 3.3255e-1 (2.32e-2) − | 3.6140e-1 (2.28e-2) |
| NN2 | 2 | 401 | 7.4992e-2 (4.36e-3) − | 3.5050e-1 (2.57e-2) = | 3.2489e-1 (1.98e-2) − | 3.4757e-1 (1.12e-2) − | 3.5029e-1 (2.33e-2) = | 3.4299e-1 (1.16e-2) − | 3.5135e-1 (2.10e-2) = | 3.5110e-1 (2.73e-2) |
| NN3 | 2 | 521 | 6.4773e-2 (3.91e-3) − | 2.9677e-1 (1.44e-2) = | 2.9685e-1 (2.56e-2) = | 2.9865e-1 (9.87e-3) = | 2.9341e-1 (1.61e-2) − | 2.9523e-1 (1.72e-2) = | 2.9952e-1 (1.95e-2) = | 2.9900e-1 (1.73e-2) |
| SR1 | 2 | 1024 | 0.0000e+0 (0.00e+0) − | 2.4202e-1 (1.43e-2) − | 8.7834e-2 (2.78e-3) − | 2.9023e-1 (2.30e-2) − | 4.3042e-2 (6.76e-2) − | 1.6190e-1 (3.92e-2) − | 2.7105e-1 (9.55e-3) − | 2.9518e-1 (2.03e-2) |
| SR2 | 2 | 5120 | 1.5965e-1 (1.08e-2) − | 1.6079e-1 (9.36e-3) − | 1.6120e-1 (9.12e-3) − | 2.3080e-1 (1.97e-2) − | 7.1402e-3 (2.26e-2) − | 8.1249e-2 (2.53e-2) − | 2.1061e-1 (1.01e-2) − | 2.7769e-1 (8.64e-3) |
| $+/-/=$ | | | 0/5/0 | 1/6/3 | 0/4/1 | 0/7/3 | 0/7/1 | 1/8/1 | 1/6/3 | $--$ |
| *Best*/*All* | | | 0/10 | 2/10 | 0/10 | 0/10 | 0/10 | 0/10 | 2/10 | 6/10 |

† "NaN" indicates that the algorithm fails to obtain any feasible solutions.