Supplementary File for "A Line Complex-Based Evolutionary Algorithm for Many-Objective Optimization"

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I. A. Sensitivity Analysis of the Parameter δ

In this paper, the maximum number of generations $\hat{g}$ is set to 1000. Here, we set a series of values of δ, ranging from 0 to 1000, to study its impact to NSGA-III/LCD, i.e., $\delta \in 0, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000$. In addition, we adopt 15 objective DTLZ1 and WFG1 benchmark problems to test the performance of NSGA-III/LCD. Except for changing the value of δ, other parameter settings remain the same as introduced in Section IV. The mean IGD results of NSGA-III/LCD with different settings of δ on 15 objective DTLZ2 and WFG1 problems are listed in Fig. S-1. From Fig. S-1, we can see that NSGA-III/LCD performs better when δ is between $0.1\hat{g}$ to $0.4\hat{g}$. The results indicate that the first stage is necessary since NSGA-III/LCD without the first stage ($\delta = 0$) shows worse performance than NSGA-III/LCD with both the first and second stages, implying the effectiveness of the proposed two-stage NSGA-III/LCD in tackling MaOPs. But the first stage is not suggested to last too long, because when $\delta > 0.4\hat{g}$ NSGA-III/LCD did not perform well. Therefore, δ is recommended to be in the range $[0.1\hat{g}, 0.4\hat{g}]$. However, to avoid unexpected performance changes, boundary values are not suggested, and this paper sets δ to $0.3\hat{g}$ which means the first stage lasts for the first $0.3\hat{g}$ generations, and the second stage is carried out for the following $0.7\hat{g}$ generations.

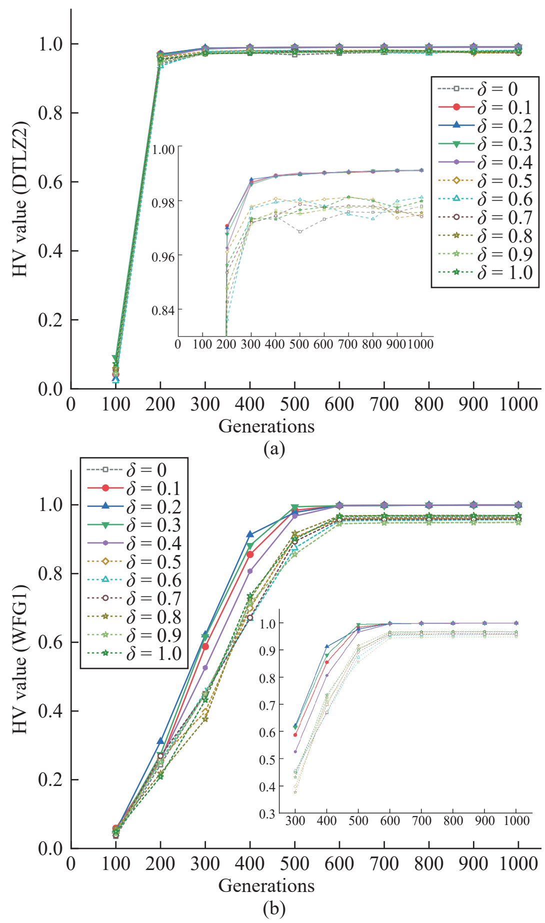

Fig. S-1. Sensitivity analysis of the parameter δ on 15 objective DTLZ2 and WFG1 problems, based on the average HV results.

II. Validation of the Line Complex

In order to observe the impact of the line complex, we designed an experiment to compare NSGA-III with the version that only integrates line complex into NSGA-III framework, named NSGA-III/LC. Test problems are DTLZ1–DTLZ7, WFG1-WFG9, SDTLZ1-SDTLZ2 and CDTLZ2 with 5-, 10-, and 15-objectives. The statistical results of the HV indicator values are shown in Table S-I, where the best results are highlighted. As shown in Table S-I, NSGA-III/LC wins 37 times out of 57 instances, and 19 of them exhibit significant improvement while 23 results are statistically equivalent to those obtained by NSGA-III.

To further improve the effectiveness of our method, a new distance function is designed for the line complex-based strategy. Experimental results in Table S-II show that, compared with NSGA-III and NSGA-III/LC, NSGA-III/LCD has been enhanced by using the distance function. Its HV indicator has a significant improvement on 26 test instances compared to NSGA-III and 20 test instances significantly better than NSGA-III/LC.

III. Validation of the Distance Function

In order to verify the differences and advantages between the

proposed distance function and the existing ones, such as that used in KnEA (The Euclidean distance, L_2-norm-based distance), BiGE (the Manhattan distance, L_1-norm-based distance) and Two_Arch2 ($L_{1/M}$-norm-based distance), experiments are also deployed to compare the HV values of NSGA-

TABLE S-I
HV VALUES OF NSGA-III AND NSGA-III/LC

Problem	M	NSGA-III	NSGA-III/LC
DTLZ1	5	9.7982e−1 (1.19e−4) ≈	**9.7988e−1 (1.51e−4)**
	10	9.9223e−1 (3.31e−2) −	**9.9966e−1 (1.89e−5)**
	15	**9.9954e−1 (1.39e−3) +**	9.9151e−1 (3.03e−2)
DTLZ2	5	8.1258e−1 (3.73e−4) ≈	**8.1276e−1 (3.78e−4)**
	10	9.6785e−1 (7.46e−3) −	**9.7299e−1 (8.98e−3)**
	15	9.7962e−1 (8.91e−3) −	**9.8524e−1 (8.60e−3)**
DTLZ3	5	**8.0988e−1 (3.38e−3) ≈**	8.0966e−1 (2.79e−3)
	10	**9.0893e−1 (2.16e−1) +**	5.4181e−1 (4.64e−1)
	15	**9.0183e−1 (2.14e−1) +**	0.0000e+0 (0.00e+0)
DTLZ4	5	8.1224e−1 (3.96e−4) ≈	**8.1252e−1 (5.01e−4)**
	10	9.6785e−1 (8.05e−3) −	**9.7296e−1 (9.29e−3)**
	15	9.8873e−1 (4.23e−3) −	**9.9265e−1 (2.72e−3)**
DTLZ5	5	1.1288e−1 (7.52e−3) ≈	**1.1466e−1 (2.59e−3)**
	10	**9.1168e−2 (1.93e−3) +**	4.5946e−2 (3.81e−2)
	15	**8.8362e−2 (3.09e−3) +**	0.0000e+0 (0.00e+0)
DTLZ6	5	**1.0624e−1 (8.26e−3) +**	9.6446e−2 (7.93e−3)
	10	**8.8902e−2 (8.92e−3) +**	0.0000e+0 (0.00e+0)
	15	**8.6386e−2 (2.03e−2) +**	0.0000e+0 (0.00e+0)
DTLZ7	5	2.5777e−1 (2.94e−3) ≈	**2.5956e−1 (2.44e−3)**
	10	1.8591e−1 (5.30e−3) ≈	**1.8633e−1 (1.40e−3)**
	15	8.7963e−2 (3.83e−2) −	**1.5416e−1 (2.62e−2)**
CDTLZ2	5	**9.9937e−1 (3.47e−5) +**	9.9930e−1 (2.13e−5)
	10	9.9983e−1 (3.64e−4) −	**1.0000e+0 (9.23e-7)**
	15	9.9972e−1 (9.87e−4) −	**1.0000e+0 (0.00e+0)**
SDTLZ1	5	9.7980e−1 (3.01e−4) ≈	**9.7987e−1 (1.19e−4)**
	10	9.9605e−1 (1.29e−2) ≈	**9.9966e−1 (1.56e−5)**
	15	9.9429e−1 (2.80e−3) −	**9.9631e−1 (1.58e−2)**
SDTLZ2	5	8.1252e−1 (4.86e−4) ≈	**8.1255e−1 (2.88e−4)**
	10	9.6847e−1 (4.43e−3) −	**9.7417e−1 (3.27e−3)**
	15	9.8692e−1 (5.23e−3) −	**9.9272e−1 (1.80e−3)**
WFG1	5	9.9833e−1 (1.36e−4) −	**9.9855e−1 (1.16e−4)**
	10	9.9928e−1 (2.68e−4) −	**9.9989e−1 (7.80e−5)**
	15	9.9989e−1 (1.56e−4) ≈	**9.9991e−1 (9.88e−5)**
WFG2	5	9.9665e−1 (4.63e−4) ≈	**9.9692e−1 (4.23e−4)**
	10	9.9802e−1 (1.04e−3) ≈	**9.9827e−1 (5.46e−4)**
	15	9.9749e−1 (1.70e−3) −	**9.9914e−1 (5.66e−4)**
WFG3	5	**9.7499e−1 (2.09e−3) +**	1.7447e−1 (1.22e−2)
	10	**9.0941e−1 (4.19e−3) +**	3.2695e−2 (2.24e−2)
	15	**7.8659e−1 (4.37e−2) +**	0.0000e+0 (0.00e+0)
WFG4	5	8.0880e−1 (8.16e−4) ≈	**8.0893e−1 (8.07e−4)**
	10	9.5650e−1 (1.07e−2) −	**9.6045e−1 (3.63e−3)**
	15	**9.8595e−1 (4.27e−3) +**	9.7840e−1 (4.72e−3)
WFG5	5	7.6143e−1 (3.65e−4) ≈	**7.6150e−1 (3.93e−4)**
	10	9.0351e−1 (3.05e−4) −	**9.0389e−1 (3.21e−4)**
	15	**9.1652e−1 (2.28e−3) +**	9.1498e−1 (4.98e−4)
WFG6	5	**7.4698e−1 (1.40e−2) ≈**	7.4464e−1 (1.50e−2)
	10	8.8144e−1 (2.03e−2) ≈	**8.8269e−1 (1.40e−2)**
	15	**8.9185e−1 (2.81e−2) ≈**	8.8905e−1 (2.09e−2)
WFG7	5	8.0965e−1 (4.77e−4) ≈	**8.0985e−1 (6.36e−4)**
	10	9.6193e−1 (1.50e−2) −	**9.6306e−1 (8.91e−3)**
	15	8.9175e−1 (2.42e−2) −	**9.8519e−1 (5.06e−3)**
WFG8	5	6.9706e−1 (3.24e−3) −	**6.9941e−1 (1.97e−3)**
	10	**8.7329e−1 (1.53e−2) ≈**	8.6748e−1 (1.00e−2)
	15	**9.2592e−1 (2.34e−2) +**	9.0844e−1 (1.23e−2)
WFG9	5	7.6576e−1 (3.96e−3) ≈	**7.6585e−1 (3.30e−3)**
	10	**8.8920e−1 (3.60e−2) ≈**	8.7829e−1 (4.94e−2)
	15	8.9077e−1 (7.43e−2) ≈	**8.9698e−1 (6.64e−2)**
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"+", "−" and "≈" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LC, respectively.

III/LC with L_1-norm-, L_2-norm-, $L_{1/M}$-norm- based distances and the proposed distances $\mathcal{D}$ on DTLZ1 with 2-, 3-, 5-, 10- and 15- objectives. The statistical results are shown in Table S-III.

As can be seen from Table S-III, the proposed distance $\mathcal{D}$ evidently outperforms L_1-norm-, L_2-norm-, $L_{1/M}$-norm-based distances on DTLZ1 with 2, 3, 10 and 15 objectives. From the comparisons above, it can be empirically concluded that the proposed distance $\mathcal{D}$ is more effective than other distance functions in defining the distance on many-objective optimization problems.

E. EXPERIMENTAL COMPARISONS OF CONVERGENCE PROFILES

Experimental comparisons of the convergence profiles of HV indicator values obtained with and without the line complex-based distance are performed. Specifically, the median HV values among 30 runs obtained by the algorithms on 15-objective WFG6 and WFG8 problems are illustrated in Fig. S-2. The HV values obtained by the algorithms on 15-objective DTLZ7 and SDTLZ2 test problems are illustrated in Fig. S-3. The red lines denote the results obtained by NSGA-III, while blue ones refer to the results by NSGA-III/LCD. The results reveal that, with the proposed line complex-based strategy, convergence is improved and better HV values are obtained.

To further observe the convergence profiles of the proposed algorithm, Fig. S-4 present IGD values obtained by NSGA-III/LCD on 5-, 10-, 15-objective DTLZ1 and CDTLZ2 test problems. The experimental results show that NSGA-III/LCD converges to the Pareto front effectively on these problems. This means the proposed line complex strategy improves the convergence in dealing with high dimensional test problems effectively.

In summary, the experimental results demonstrate that the proposed line complex-based strategy can effectively improves both convergence and diversity of basic algorithm (NSGA-III) on DTLZ and WFG test problems.

IV. PLOT RESULTS ON THREE-OBJECTIVE PROBLEMS

Comparison of all methods (with the same experimental settings, the population size of all the algorithms is set to 92) on 3-objective DTLZ1-7, WFG1-9, SDTLZ1-2 and CDTLZ are shown in Fig. S-5. It can be seen that, NSGA-III/LCD can obtain better distributed solutions in most cases, although it does not perform very well on DTLZ7 and WFG3. In the 3-objective optimization experiment, we found that NSGA-II/SDR does not perform well. It is because that the SDR strategy does not adopt Pareto dominance to regard solutions. Candidate solutions that are worse in all objectives compared to another solution (i.e., the candidate solutions that are Pareto-dominated) may be regarded as nondominated by SDR, which is not desired. On multi-objective optimization problems (i.e., 3-objective test problems) a considerable number of solutions are Pareto-dominated which makes SDR perform poorly. However, on many-objective optimization problems (MOPs with more than three objectives), only a few candidate solutions in the population are Pareto-dominated, which have little influence on the performance of NSGA-II/SDR.

TABLE S-II
HV VALUES OF NSGA-III, NSGA-III/LC, AND NSGA-III/LCD

Problem	M	NSGA-III	NSGA-III/LC	NSGA-III/LCD
DTLZ1	5	9.7982e−1 (1.19e−4) ≈	**9.7988e−1 (1.51e−4)** ≈	9.7987e−1 (1.61e−4)
	10	9.9223e−1 (3.31e−2) −	9.9966e−1 (1.89e−5) −	**9.9969e−1 (1.71e−5)**
	15	9.9954e−1 (1.39e−3) −	9.9151e−1 (3.03e−2) −	**9.9994e−1 (1.42e−5)**
DTLZ2	5	8.1258e−1 (3.73e−4) −	**8.1276e−1 (3.78e−4)** ≈	8.1285e−1 (2.89e−4)
	10	9.6785e−1 (7.46e−3) +	**9.7299e−1 (8.98e−3)** +	9.6677e−1 (1.02e−2)
	15	9.7962e−1 (8.91e−3) −	**9.8524e−1 (8.60e−3)** ≈	9.8517e−1 (4.99e−3)
DTLZ3	5	8.0988e−1 (3.38e−3) ≈	8.0966e−1 (2.79e−3) ≈	**8.1081e−1 (1.29e−3)**
	10	9.0893e−1 (2.16e−1) ≈	5.4181e−1 (4.64e−1) −	**9.6196e−1 (1.99e−2)**
	15	**9.0183e−1 (2.14e−1)** ≈	0.0000e+0 (0.00e+0) −	7.5910e−1 (3.76e−1)
DTLZ4	5	8.1224e−1 (3.96e−4) −	8.1252e−1 (5.01e−4) ≈	**8.1270e−1 (3.39e−4)**
	10	9.6785e−1 (8.05e−3) −	9.7296e−1 (9.29e−3) −	**9.7406e−1 (6.20e−4)**
	15	9.8873e−1 (4.23e−3) −	**9.9265e−1 (2.72e−3)** +	9.9130e−1 (1.02e−4)
DTLZ5	5	1.1288e−1 (7.52e−3) −	1.1466e−1 (2.59e−3) ≈	**1.1618e−1 (1.18e−3)**
	10	**9.1168e−2 (1.93e−3)** +	4.5946e−2 (3.81e−2) −	8.8170e−2 (2.50e−3)
	15	**8.8362e−2 (3.09e−3)** +	0.0000e+0 (0.00e+0) −	8.1804e−2 (8.86e−3)
DTLZ6	5	**1.0624e−1 (8.26e−3)** ≈	9.6446e−2 (7.93e−3) −	1.0151e−1 (8.84e−3)
	10	**8.8902e−2 (8.92e−3)** +	0.0000e+0 (0.00e+0) ≈	4.5455e−3 (2.03e−2)
	15	**8.6386e−2 (2.03e−2)** +	0.0000e+0 (0.00e+0) ≈	9.1064e−3 (2.80e−2)
DTLZ7	5	2.5777e−1 (2.94e−3) +	**2.5956e−1 (2.44e−3)** +	2.5472e−1 (4.36e−3)
	10	1.8591e−1 (5.30e−3) ≈	1.8633e−1 (1.40e−3) ≈	**1.8816e−1 (5.12e−3)**
	15	8.7963e−2 (3.83e−2) −	**1.5416e−1 (2.62e−2)** +	1.5290e−1 (6.11e−3)
CDTLZ2	5	**9.9937e−1 (3.47e−5)** +	9.9930e−1 (2.13e−5) −	9.9935e−1 (4.37e−5)
	10	9.9983e−1 (3.64e−4) −	**1.0000e+0 (9.23e-7)** ≈	**1.0000e+0 (2.48e−6)**
	15	9.9972e−1 (9.87e−4) ≈	**1.0000e+0 (0.00e+0)** +	9.9994e−1 (1.33e−4)
SDTLZ1	5	9.7980e−1 (3.01e−4) ≈	**9.7987e−1 (1.19e−4)** ≈	9.7985e−1 (1.57e−4)
	10	9.9605e−1 (1.29e−2) ≈	9.9966e−1 (1.56e−5) −	**9.9967e−1 (3.42e−5)**
	15	9.9429e−1 (2.80e−3) −	9.9631e−1 (1.58e−2) −	**9.9832e−1 (2.20e−3)**
SDTLZ2	5	8.1252e−1 (4.86e−4) ≈	8.1255e−1 (2.88e−4) ≈	**8.1271e−1 (4.00e−4)**
	10	9.6847e−1 (4.43e−3) ≈	**9.7417e−1 (3.27e−3)** +	9.6497e−1 (1.59e−2)
	15	9.8692e−1 (5.23e−3) ≈	**9.9272e−1 (1.80e−3)** +	9.8874e−1 (3.67e−3)
WFG1	5	9.9833e−1 (1.36e−4) −	**9.9855e−1 (1.16e−4)** ≈	9.9850e−1 (2.29e−4)
	10	9.9928e−1 (2.68e−4) −	**9.9989e−1 (7.80e−5)** +	9.9977e−1 (1.28e−4)
	15	9.9989e−1 (1.56e−4) −	9.9991e−1 (9.88e−5) −	**9.9999e−1 (9.30e−6)**
WFG2	5	9.9665e−1 (4.63e−4) −	9.9692e−1 (4.23e−4) ≈	**9.9695e−1 (4.10e−4)**
	10	9.9802e−1 (1.04e−3) ≈	9.9827e−1 (5.46e−4) ≈	9.9848e−1 (9.78e−4)
	15	9.9749e−1 (1.70e−3) −	**9.9914e−1 (5.66e−4)** +	9.9856e−1 (7.80e−4)
WFG3	5	9.7499e−1 (2.09e−3) −	1.7447e−1 (1.22e−2) −	**9.7619e−1 (2.12e−3)**
	10	9.0941e−1 (4.19e−3) −	3.2695e−2 (2.24e−2) −	**9.1396e−1 (4.92e−3)**
	15	7.8659e−1 (4.37e−2) ≈	0.0000e+0 (0.00e+0) −	**7.9194e−1 (2.52e−2)**
WFG4	5	8.0880e−1 (8.16e−4) ≈	**8.0893e−1 (8.07e−4)** ≈	8.0856e−1 (9.05e−4)
	10	9.5650e−1 (1.07e−2) −	9.6045e−1 (3.63e−3) ≈	**9.6178e−1 (2.63e−3)**
	15	9.8595e−1 (4.27e−3) −	9.7840e−1 (4.72e−3) −	**9.8774e−1 (3.12e−3)**
WFG5	5	7.6143e−1 (3.65e−4) −	7.6150e−1 (3.93e−4) ≈	**7.6167e−1 (3.29e−4)**
	10	9.0351e−1 (3.05e−4) −	9.0389e−1 (3.21e−4) ≈	**9.0395e−1 (2.45e−4)**
	15	9.1652e−1 (2.28e−3) −	9.1498e−1 (4.98e−4) −	**9.1672e−1 (1.82e−3)**
WFG6	5	7.4698e−1 (1.40e−2) ≈	7.4464e−1 (1.50e−2) ≈	**7.5242e−1 (1.29e−2)**
	10	8.8144e−1 (2.03e−2) ≈	8.8269e−1 (1.40e−2) ≈	**8.8787e−1 (1.20e−2)**
	15	8.9185e−1 (2.81e−2) ≈	8.8905e−1 (2.09e−2) ≈	**8.9553e−1 (1.85e−2)**
WFG7	5	8.0965e−1 (4.77e−4) ≈	**8.0985e−1 (6.36e−4)** ≈	8.0960e−1 (4.62e−4)
	10	9.6193e−1 (1.50e−2) −	9.6306e−1 (8.91e−3) −	**9.6704e−1 (1.08e−3)**
	15	8.9175e−1 (2.42e−2) −	**9.8519e−1 (5.06e−3)** +	9.1386e−1 (1.22e−2)
WFG8	5	6.9706e−1 (3.24e−3) −	6.9941e−1 (1.97e−3) ≈	**6.9980e−1 (2.03e−3)**
	10	8.7329e−1 (1.53e−2) ≈	8.6748e−1 (1.00e−2) −	**8.7653e−1 (1.56e−2)**
	15	9.2592e−1 (2.34e−2) ≈	9.0844e−1 (1.23e−2) −	**9.3671e−1 (2.25e−2)**
WFG9	5	7.6576e−1 (3.96e−3) ≈	7.6585e−1 (3.30e−3) ≈	**7.6710e−1 (4.61e−3)**
	10	**8.8920e−1 (3.60e−2)** ≈	8.7829e−1 (4.94e−2) ≈	8.6992e−1 (6.32e−2)
	15	8.9077e−1 (7.43e−2) ≈	8.9698e−1 (6.64e−2) ≈	**9.1345e−1 (4.42e−2)**
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"+", "−" and "≈" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LCD respectively.

TABLE S-III

AVERAGE HV VALUES OF NSGA-III/LC WITH L_1-, L_2-, $L_{1/M}$-NORM- BASED DISTANCES AND $\mathcal{D}$ ON DTLZ1 WITH 2-, 3-, 5-, 10-, AND 15-OBJECTIVES

M	L_1-norm	L_2-norm	$L_{1/M}$-norm	$\mathcal{D}$
2	5.8204e−1 (1.33e−4) rank 3	5.8203e−1 (1.58e−4) rank 4	5.8208e−1 (1.37e−4) rank 2	**5.8212e−1 (7.24e−5) rank 1**
3	8.4153e−1 (2.03e−4) rank 3	8.4151e−1 (3.04e−4) rank 4	8.4156e−1 (1.60e−4) rank 2	**8.4163e−1 (9.26e−5) rank 1**
5	9.7789e−1 (1.51e−4) rank 4	9.7982e−1 (1.27e−4) rank 4	9.7684e−1 (1.25e−4) rank 3	**9.7987e−1 (1.61e−4) rank 1**
10	9.8319e−1 (2.67e−2) rank 4	9.9223e−1 (3.31e−2) rank 2	9.9962e−1 (4.85e−5) rank 3	**9.9969e−1 (1.71e−5) rank 1**
15	9.9853e−1 (2.27e−3) rank 3	9.9954e−1 (1.39e−3) rank 4	9.9981e−1 (7.61e−4) rank 2	**9.9994e−1 (1.42e−5) rank 1**
Average rank	**3.8**	**2.6**	**2.4**	**1.2**

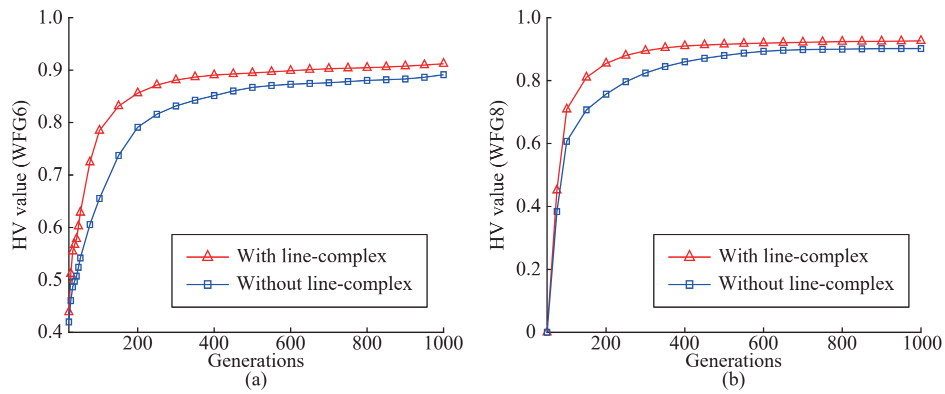


Fig. S-2. HV values of the results obtained by the algorithms on 15-objective (a) WFG6 and (b) WFG8 test problems with and without the line complex over 1000 generations.

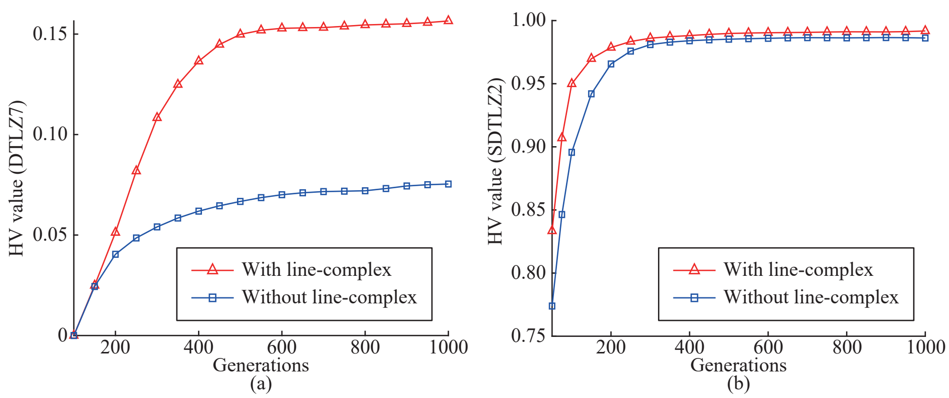


Fig. S-3. HV values of the results obtained by the algorithms on 15-objective (a) DTLZ7 and (b) SDTLZ2 test problems with and without the line complex over 1000 generations.

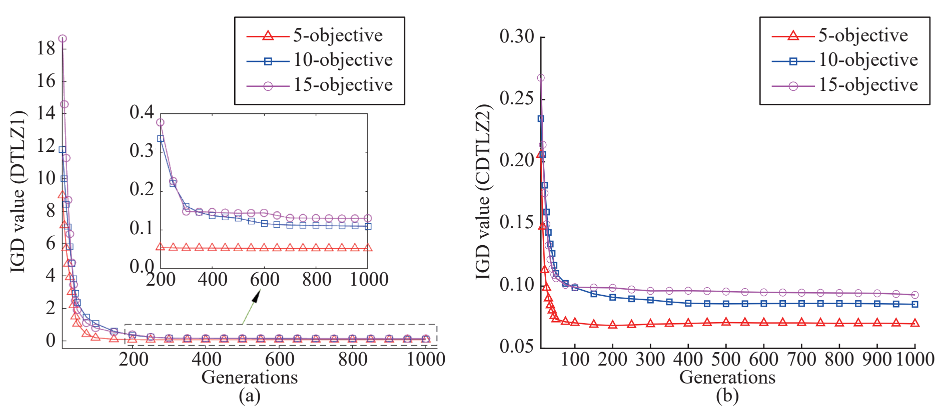
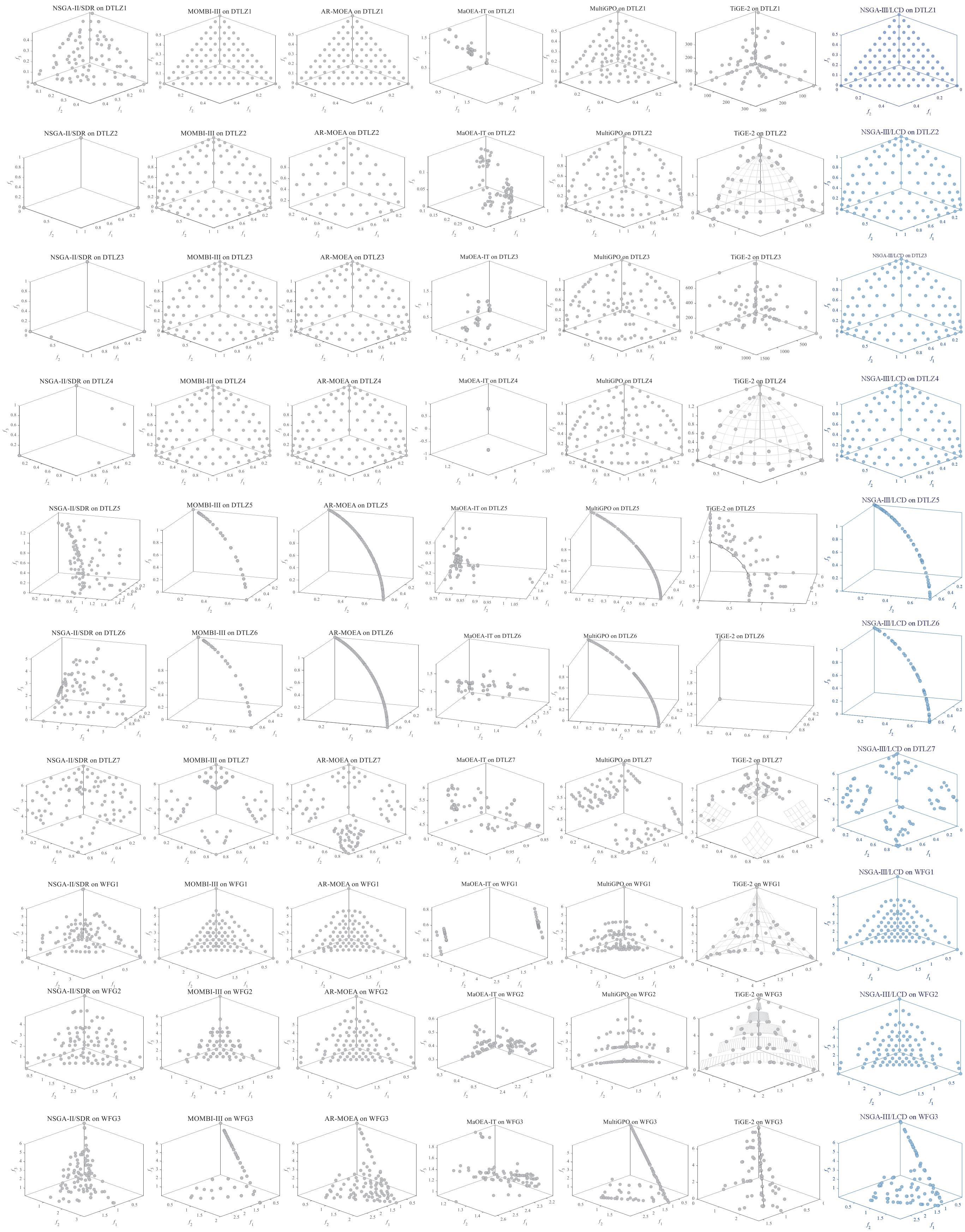


Fig. S-4. Convergence profiles of IGD values obtained by NSGA-III/LCD on 5-, 10-, 15-objective DTLZ1 with regular PFs and 5-, 10-, 15-objective CDTLZ2 with irregular PFs, all results are averaged over 30 runs.


NSGA-II/SDR on DTLZ1
MOMBI-III on DTLZ1
AR-MOEA on DTLZ1
MaOEA-IT on DTLZ1
MultiGPO on DTLZ1
TiGE-2 on DTLZ1
NSGA-III/LCD on DTLZ1
NSGA-II/SDR on DTLZ2
MOMBI-III on DTLZ2
AR-MOEA on DTLZ2
MaOEA-IT on DTLZ2
MultiGPO on DTLZ2
TiGE-2 on DTLZ2
NSGA-III/LCD on DTLZ2
NSGA-II/SDR on DTLZ3
MOMBI-III on DTLZ3
AR-MOEA on DTLZ3
MaOEA-IT on DTLZ3
MultiGPO on DTLZ3
TiGE-2 on DTLZ3
NSGA-III/LCD on DTLZ3
NSGA-II/SDR on DTLZ4
MOMBI-III on DTLZ4
AR-MOEA on DTLZ4
MaOEA-IT on DTLZ4
MultiGPO on DTLZ4
TiGE-2 on DTLZ4
NSGA-III/LCD on DTLZ4
NSGA-II/SDR on DTLZ5
MOMBI-III on DTLZ5
AR-MOEA on DTLZ5
MaOEA-IT on DTLZ5
MultiGPO on DTLZ5
TiGE-2 on DTLZ5
NSGA-III/LCD on DTLZ5
NSGA-II/SDR on DTLZ6
MOMBI-III on DTLZ6
AR-MOEA on DTLZ6
MaOEA-IT on DTLZ6
MultiGPO on DTLZ6
TiGE-2 on DTLZ6
NSGA-III/LCD on DTLZ6
NSGA-II/SDR on DTLZ7
MOMBI-III on DTLZ7
AR-MOEA on DTLZ7
MaOEA-IT on DTLZ7
MultiGPO on DTLZ7
TiGE-2 on DTLZ7
NSGA-III/LCD on DTLZ7
NSGA-II/SDR on WFG1
MOMBI-III on WFG1
AR-MOEA on WFG1
MaOEA-IT on WFG1
MultiGPO on WFG1
TiGE-2 on WFG1
NSGA-III/LCD on WFG1
NSGA-II/SDR on WFG2
MOMBI-III on WFG2
AR-MOEA on WFG2
MaOEA-IT on WFG2
MultiGPO on WFG2
TiGE-2 on WFG3
NSGA-III/LCD on WFG2
NSGA-II/SDR on WFG3
MOMBI-III on WFG3
AR-MOEA on WFG3
MaOEA-IT on WFG3
MultiGPO on WFG3
TiGE-2 on WFG3
NSGA-III/LCD on WFG3

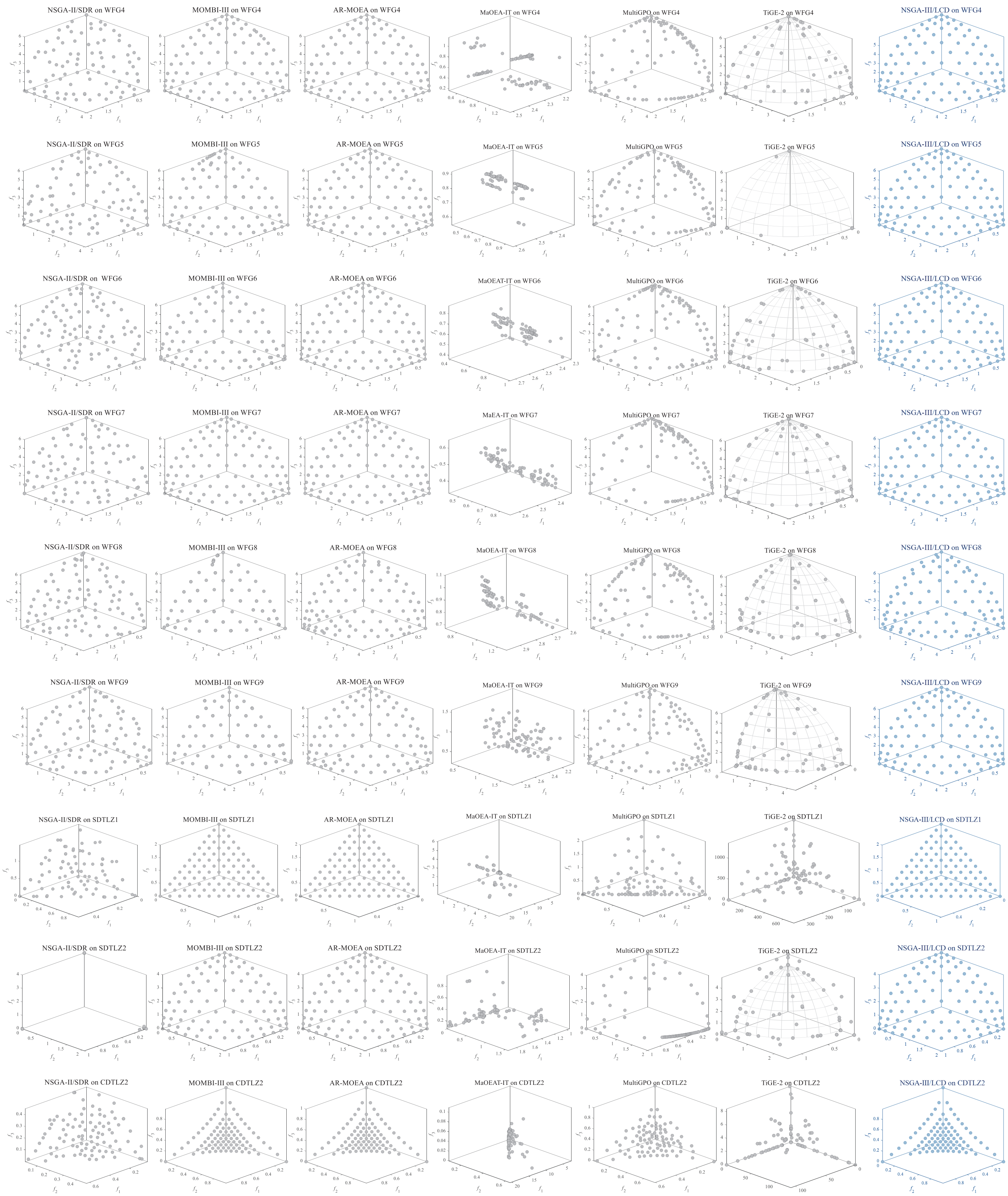


Fig. S-5. Plot results for each algorithm on 3-objective DTLZ1−7, WFG1−9, SDTLZ1−2 and CDTLZ.