Supplementary Material of "Finite-Time Attack Detection and Secure State Estimation for Cyber-Physical Systems"

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Proof of Lemma 2

Proof: It is clear that the observability of (A,C) is equivalent to

$$\operatorname{rank}\begin{bmatrix}\lambda I_n - A\\ C\end{bmatrix} = n, \quad \forall \lambda \in \mathbb{C}. \tag{1}$$

Note that

$$\begin{aligned}\operatorname{rank}\begin{bmatrix}\lambda I_n - A\\ C\end{bmatrix} &= \operatorname{rank}\begin{bmatrix}P & 0\\ 0 & I_m\end{bmatrix}\begin{bmatrix}\lambda I_n - A\\ C\end{bmatrix}P^{-1}\\ &= \operatorname{rank}\begin{bmatrix}\lambda I_{n-q} - \bar{A}_{11}\\ -\bar{A}_{21}\\ C_1 N\end{bmatrix} + q.\end{aligned} \tag{2}$$

Thus, we have that the observability of (A,C) is equivalent to $(\bar{A}_{11}, \bar{C}_1)$ is observable.

$$\operatorname{rank}\begin{bmatrix}\lambda I_{n-q} - \bar{A}_{11}\\ \bar{C}_1\end{bmatrix} = n-q, \quad \forall \lambda \in \mathbb{C}. \tag{3}$$

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Proof of Theorem 2

Proof: Note that

$$\begin{aligned}[T \quad F^\tau T] &= \begin{bmatrix}I_{n-q} & F_1^\tau\\ I_{n-q} & F_2^\tau\end{bmatrix}\\ &= \begin{bmatrix}I_{n-q} & 0\\ I_{n-q} & -I_{n-q}\end{bmatrix}\begin{bmatrix}I_{n-q} & F_1^\tau\\ 0 & F_1^\tau - F_2^\tau\end{bmatrix}\end{aligned} \tag{4}$$

which indicates that

$$\begin{aligned}\det[T \quad F^\tau T] &= (-1)^{n-q}\det[F_1^\tau - F_2^\tau]\\ &= (-1)^{n-q}\det[F_1^\tau]\det\left[I_{n-q} - F_1^{-\tau}F_2^\tau\right].\end{aligned} \tag{5}$$

Since all the eigenvalues of F_1 are nonzero, we have that $\det[F_1^\tau] \neq 0$. Note that $\det[I_{n-q} - F_1^{-\tau}F_2^\tau]$ is an analytic function of τ and $\det[I_{n-q} - F_1^{-\tau}F_2^\tau] = \det[I_{n-q} - F_1^{-\tau}\sigma^\tau F_2^\tau \sigma^{-\tau}] \to 1$ as $\tau \to \infty$, indicating that it only has some isolated zeros. Since $\det[I_{n-q} - F_1^{-\tau}F_2^\tau] = 0$ when $\tau = 0$, there must be a τ^* such that $\det[I_{n-q} - F_1^{-\tau}F_2^\tau] \neq 0$ for all $\tau \in (0, \tau^*)$. ■